Sample Test 4

1. True or False: Clearly indicate your choice.

(a) T F \[ \int_0^2 \int_1^5 f(x, y) \, dx \, dy = \int_1^5 \int_0^2 f(x, y) \, dx \, dy. \]

(b) T F Let \( a \) = the area of the region \( R \) in the \( xy \)-plane. If \( f(x, y) = k \) for all \( x \) and \( y \) in \( R \), then \( \int \int_R f(x, y) \, dA = ka. \)

(c) T F The volume of the sphere \( x^2 + y^2 + z^2 = 1 \) is given by
\[
V = 8 \int_0^1 \int_0^1 \sqrt{1 - x^2 - y^2} \, dx \, dy
\]

(d) T F \[ \int_0^2 \int_0^{2\pi} \int_0^{3\sin \phi} \rho^2 \sin \phi \, d\rho \, d\phi \, d\phi \]

2. \[ \int_0^1 \int_1^3 \int \, dx \, dy = \]
    (a) 2 (b) 5 (c) 4 (d) \( \frac{1}{2} \) (e) 10 (f) Does not Exist

3. \[ \int_0^1 \int_0^x \sqrt{1 - x^2} \, dy \, dx = \]
   (a) \( \frac{1}{2} \) (b) \( \frac{1}{3} \) (c) \( \frac{1}{4} \) (d) 1 (e) 2 (f) 2x

4. The following integral defines the volume of the region inside the sphere \( x^2 + y^2 + z^2 = a^2 \) above the \( xy \)-plane over the region \( x^2 + y^2 \leq 1 \)
\[
\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \sqrt{a^2 - x^2 - y^2} \, dy \, dx.
\]

When this is converted to polar coordinates, the resulting double integral has the form

(a) \[ \int_0^{2\pi} \int_0^1 r^2 \, d\theta \]
  (b) \[ \int_0^{2\pi} \int_0^1 r \sqrt{a^2 - r^2} \, d\theta \, dr \]
  (c) \[ \int_0^{2\pi} \int_0^1 r \, dr \, d\theta \]
  (d) \[ \int_0^{2\pi} \int_0^1 \sqrt{a^2 - r^2} \, dr \, d\theta \]
  (e) \[ \int_0^{2\pi} \int_0^1 r \sqrt{a^2 - r^2} \, dr \, d\theta \]
5. The iterated integral \( \int_{0}^{2} \int_{0}^{x^2} f(x, y) \, dy \, dx \) is equivalent to

(a) \( \int_{0}^{4} \int_{0}^{\sqrt{y}} f(x, y) \, dx \, dy \)
(b) \( \int_{0}^{4} \int_{0}^{2} f(x, y) \, dx \, dy \)
(c) \( \int_{0}^{2} \int_{0}^{\sqrt{y}} f(x, y) \, dx \, dy \)

(d) \( \int_{0}^{4} \int_{0}^{\sqrt{y}} f(x, y) \, dx \, dy \)
(e) \( \int_{0}^{2} \int_{0}^{\sqrt{y}} f(x, y) \, dx \, dy \)

6. Evaluate the following iterated integral. Exact answers only. No numerical approximations.

\( \int_{0}^{6} \int_{0}^{x} e^{y^2} \, dy \, dx \)

7. Express the following as a single iterated integral. Do not evaluate it. \(10 \text{ pts}\)

\( \int_{0}^{2} \int_{0}^{x} dy \, dx + \int_{2}^{4} \int_{0}^{4-x} dy \, dx \)

8. Set up (but do not evaluate) a double integral that gives the area of the surface on the graph of \( f(x, y) = xe^y \) over the region \( R \) consisting of all \( x \) and \( y \) such that \( 0 \leq x \leq 4 \) and \( 0 \leq y \leq 10 \).

9. Set up but do not evaluate a triple integral for the mass of the solid region \( Q \) bounded by the graphs of the equations below. Assume that density is proportional to the distance from the origin.

\( Q:\quad z = 2 - y, \quad z = 0, \quad y = 0, \quad x = 3, \quad x = 0 \)

10. Suppose \( Q \) is the region in space above the \( xy \)-plane inside the sphere \( x^2 + y^2 + z^2 = 16 \). Consider the integral

\[ \int \int \int_{Q} (x^2 + y^2) \, dV \]

which computes the moment of inertia of \( Q \) around the \( z \)-axis. \(12 \text{ pts}\)

(a) Set up (but do not evaluate) an iterated triple integral in \textbf{cylindrical coordinates} that could be used to evaluate this integral.

11. Find the volume of the region below \( z = x^2 + y^2 \) and above the region \( x^2 + y^2 \leq 1 \).

12. Use a double integral in polar coordinates to find the volume of the solid in the \textbf{first octant} \( (x \geq 0, y \geq 0, z \geq 0) \) bounded above by \( z = e^{x^2+y^2} \) and below by \( x^2 + y^2 \leq 1 \). Show your work. Exact answers only. NO NUMERICAL APPROXIMATIONS. \(4 \text{ pts}\)