More on the Alternating Series Test and Absolute vs. Conditional Convergence

Recall: (The Alternating Series Test) Suppose that for all \( n \), we have \( a_n > 0 \). The alternating series \( \sum_{n=1}^{\infty} (-1)^n a_n \) and \( \sum_{n=1}^{\infty} (-1)^{n+1} a_n \) converge if . . .

1.

2.

Thinking Question: Is condition 2 in the Alternating Series Test really necessary? Doesn’t condition 1 imply condition 2?

Answer: Consider the series \( \sum_{n=1}^{\infty} (-1)^{n+1} a_n \), where . . .
Observe:

**Alt. Series Remainder Thm:** Suppose the series $\sum_{n=1}^{\infty} (-1)^n a_n$ (or $\sum_{n=1}^{\infty} (-1)^{n+1} a_n$) satisfies the conditions in the Alternating Series Test. So by the Alt. Series Test, the series converges to some number $S = \sum_{n=1}^{\infty} (-1)^n a_n$. Then the $n$th partial sum $S_n$ is such that . . .

**Ex 1:** Observe that the alternating series $\sum_{n=1}^{\infty} \frac{(-1)^n}{\ln n}$ converges. What is the minimum partial sum $S_n$ that is guaranteed to be within 0.1 of the actual sum?
**Ex 2:** It can be shown, using the Alternating Series Test, that the series \( \sum_{n=1}^{\infty} \frac{(-1)^n(n + 2)}{n^2 + 3n + 5} \) converges. Determine the minimum partial sum \( S_n \) that is guaranteed to be within 0.1 of the actual sum.

**DO NOT COMPUTE THIS PARTIAL SUM!** (This is what computers are for.)

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**Absolute vs. Conditional Convergence**

**Ex 3:** Does the series \( \sum_{n=1}^{\infty} \frac{\cos n}{n^3} \) converge or diverge?
Thm:

Def: We say a series $\sum a_n$ is absolutely convergent if . . .

We say a series $\sum a_n$ is conditionally convergent if . . .

Ex 4: Determine whether the series converges conditionally or absolutely, or diverges.

(a) $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n}$
(b) \[ \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+1}} \]