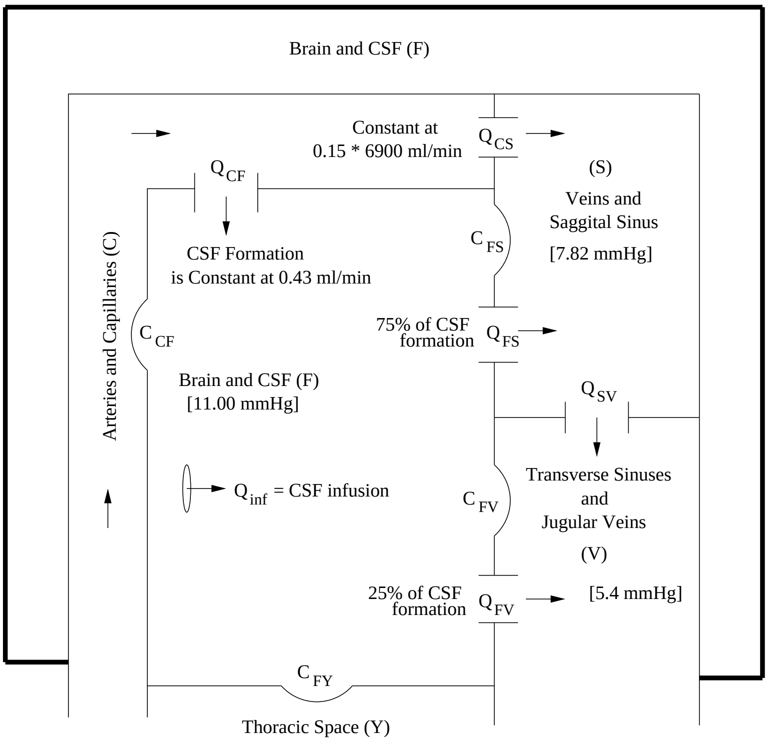


$$\begin{aligned} \frac{dV_F}{dt} &= \overline{Q}_{CF} - Z_{FS}(P_F - P_S) - Z_{FV}(P_F - P_V) && (1) \\ \frac{dV_S}{dt} &= \overline{Q}_{CS} + Z_{FS}(P_F - P_S) - Z_{SV}(P_S - P_V) && (2) \end{aligned}$$

$$\begin{aligned} C_{CF}\frac{dP_{FC}}{dt} + C_{FS}\frac{dP_{FS}}{dt} + C_{FV}\frac{dP_{FV}}{dt} + C_{FY}\frac{dP_{FY}}{dt} &= (Q_{CF} + Q_{inf}) - (Z_{FS}P_{FS} + Z_{FV}P_{FV}) && (3) \\ C_{FS}\frac{dP_{FS}}{dt} &= (Q_{CS} + Z_{FS}P_{FS}) - Z_{SV}P_{SV}, && (4) \end{aligned}$$

Now let P_V and P_C be held constant at $\overline{P}_V$ and $\overline{P}_C$ respectively, and $Q_{inf} = 0$.

$$\begin{pmatrix} C_{FS} + C_F & -C_{FS} \\ -C_{FS} & C_{FS} \end{pmatrix} \begin{pmatrix} \dot{P}_F \\ \dot{P}_S \end{pmatrix} = \begin{pmatrix} -(Z_{FS} + Z_{FV}) & Z_{FS} \\ Z_{FS} & -(Z_{FS} + Z_{SV}) \end{pmatrix} \begin{pmatrix} P_F \\ P_S \end{pmatrix} + \begin{pmatrix} \overline{Q}_{CF} + Z_{FV}\overline{P}_V \\ \overline{Q}_{CS} + Z_{SV}\overline{P}_V \end{pmatrix} \quad (5)$$

where $C_F = C_{FV} + C_{FC} + C_{FY}$.

Now let $P_F = \overline{P}_F + x(t)$ and $P_S = \overline{P}_S + y(t)$. Show that $x(t)$ and $y(t)$ satisfy

$$\begin{pmatrix} C_{FS} + C_F & -C_{FS} \\ -C_{FS} & C_{FS} \end{pmatrix} \begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} -(Z_{FS} + Z_{FV}) & Z_{FS} \\ Z_{FS} & -(Z_{FS} + Z_{SV}) \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \quad (6)$$

Show that (0,0) is the lone steady-state and it is a stable node. MAPLE.

Resistance: $Q = \dfrac{\Delta P}{R}$ (R is the resistance = inverse of fluidity).

In order to accommodate the existence of a collapsed sinus and the resistance dependence upon downstream transmural pressure we consider a Starling-like resistor of the form

$$R_{SV} = \begin{cases} q\overline{R}_{SV} & \text{if} \quad P_{FV} > P_{FV}^{max} = \frac{q-1}{m} + \overline{P}_{FV} \\ \left[1 + m(P_{FV} - \overline{P}_{FV})\right]\overline{R}_{SV} & \text{if} \quad P_{FV}^{min} \le P_{FV} \le P_{FV}^{max} \\ p\overline{R}_{SV} & \text{if} \quad P_{FV} < P_{FV}^{min} = \overline{P}_{FV} - \frac{1-p}{m} \end{cases} . \tag{7}$$

Here, $q \ge 1$ and $0 \le p \le 1$. An example of the the graph of such a resistor is given in figure 1.

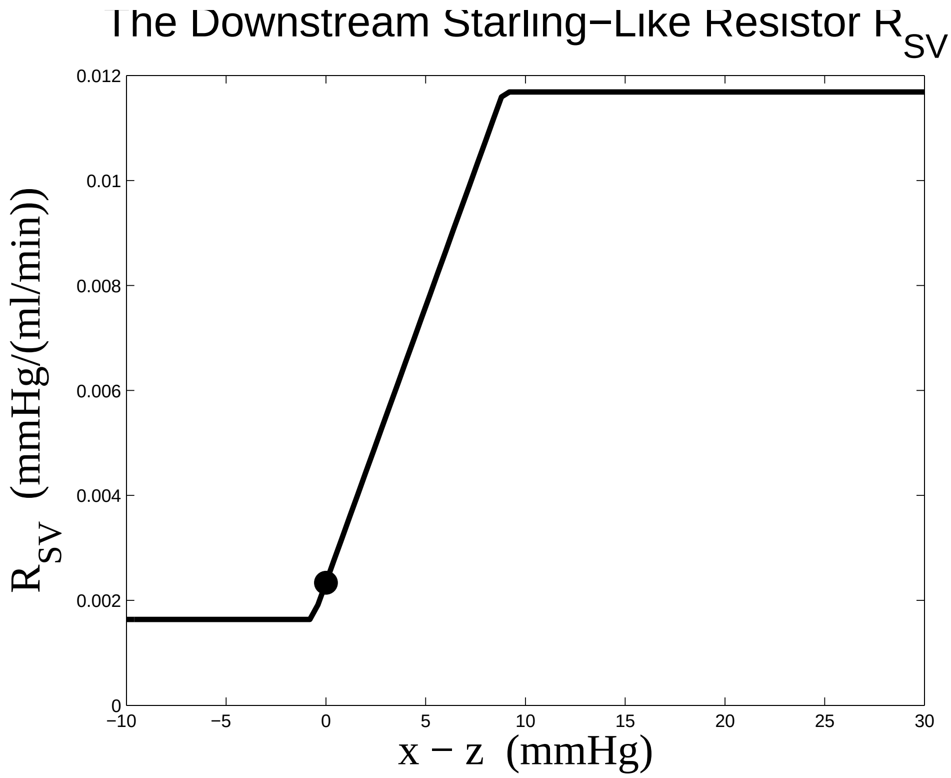


Figure 1: The downstream Starling resistor describe by equation 7. Here, $x = (P_F - \overline{P}_F)$ and $z = (P_V - \overline{P}_V)$

Let

$$x = P_F - \overline{P}_F \tag{8}$$
$$y = P_S - \overline{P}_S \tag{9}$$
$$z = P_V - \overline{P}_V \tag{10}$$

Now express R_{SV} in terms of x and y. Plug this into equation 6 and investigate the dynamics using maple.

Here are the equations in x and y.

$$(C_F + C_{FS})\dot{x} - C_{FS}\dot{y} = (Q_{CF} + Q_{inf}) - \left(\frac{(x-y) + \overline{P}_{FS}}{R_{FS}} + \frac{(x-z) + \overline{P}_{FV}}{R_{FV}} \right) \tag{11}$$

$$-C_{FS}\dot{x} + C_{FS}\dot{y} = Q_{CS} + \frac{(x-y) + \overline{P}_{FS}}{R_{FS}} - \frac{(y-z) + \overline{P}_{SV}}{R_{SV}} \tag{12}$$

$$R_{SV} = \begin{cases} q\,\overline{R}_{SV} & \text{if} \quad x > z + \frac{q-1}{m} \\ (1 + m(x-z))\,\overline{R}_{SV} & \text{if} \quad z - \frac{1-p}{m} \le x \le z + \frac{q-1}{m} \\ p\,\overline{R}_{SV} & \text{if} \quad x < z - \frac{1-p}{m} \end{cases} . \tag{13}$$

Start with $q = .2$ and $p = 10$.

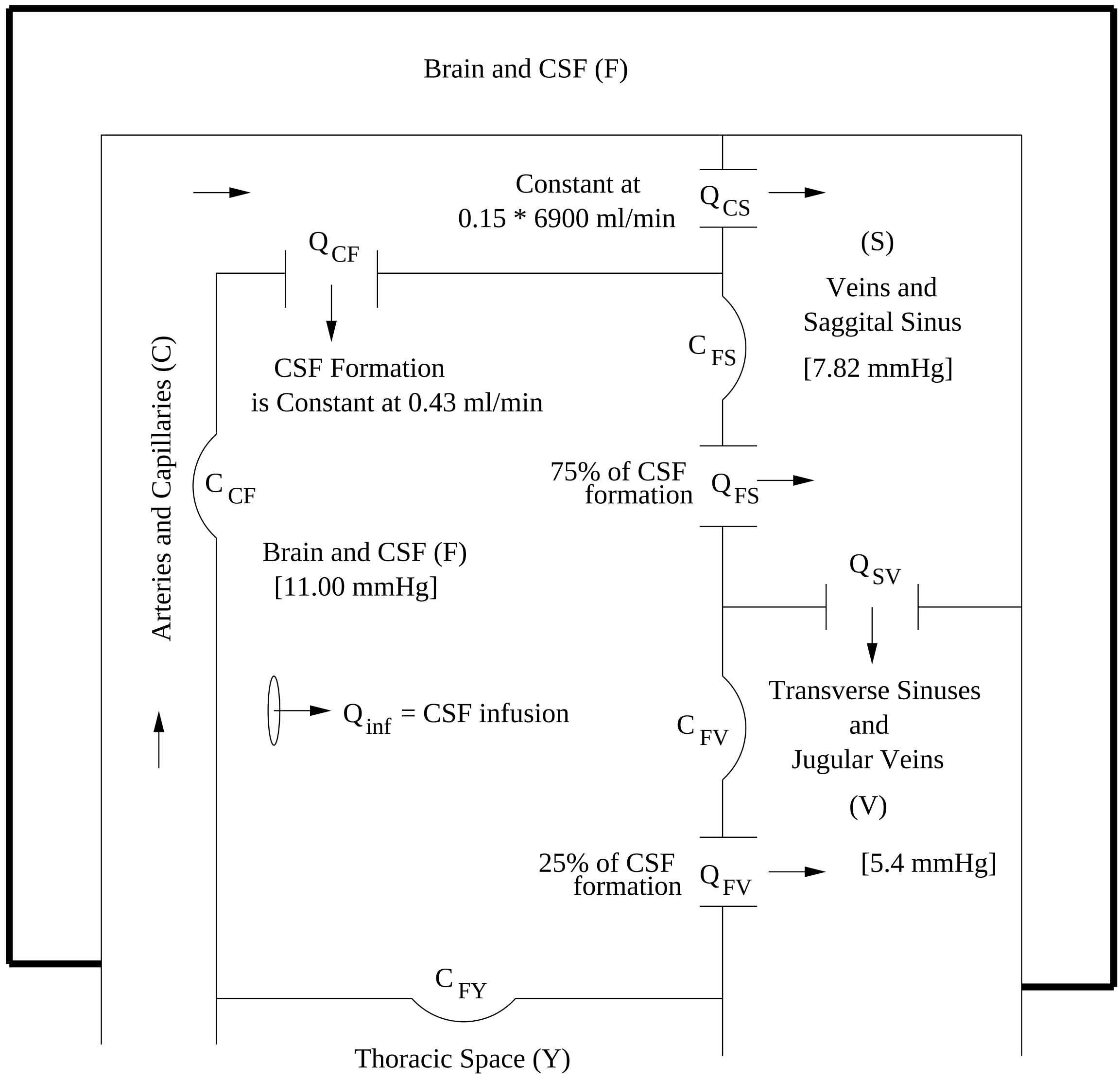
$$\overline{Q}_{CF} = .43 \quad \text{ml/min} \tag{14}$$
$$\overline{Q}_{CS} = (0.15)(6900) \quad \text{ml/min} \tag{15}$$
$$\overline{Q}_{FS} = 0.75\ \overline{Q}_{CF} \quad \text{ml/min} \tag{16}$$
$$\overline{Q}_{FV} = 0.25\ \overline{Q}_{CF} \quad \text{ml/min} \tag{17}$$
$$\overline{Q}_{SV} = \overline{Q}_{CS} + \overline{Q}_{FS} \quad \text{ml/min} \tag{18}$$

$$\overline{P}_F = 11 \quad \text{mmHg} \tag{19}$$
$$\overline{P}_S = 7.82 \quad \text{mmHg} \tag{20}$$
$$\overline{P}_V = 5.4 \quad \text{mmHg} \tag{21}$$

$$\overline{R}_{IJ} = \frac{\overline{P}_I - \overline{P}_J}{\overline{Q}_{IJ}} \tag{22}$$

Start with $q = .2$ and $p = 10$. We will vary m and see how the dynamics change for various initial conditions.

m	x(0)	y(0)	What is going on here?
.40	1	1	
.40	-1	-1	
.443	0	0	
.444	0	0	
.445	0	0	
.45	0	0	
.495	0	0	
.495	1	1	
.495	-1	-1	


Brain and CSF (F)
Constant at
0.15 * 6900 ml/min
Q CS
Q CF
(S)
Veins and
Saggital Sinus
C FS
[7.82 mmHg]
CSF Formation
is Constant at 0.43 ml/min
Arteries and Capillaries (C)
C CF
75% of CSF
formation
Q FS
Brain and CSF (F)
[11.00 mmHg]
Q SV
Transverse Sinuses
and
Jugular Veins
Q inf = CSF infusion
C FV
(V)
25% of CSF
formation
Q FV
[5.4 mmHg]
C FY
Thoracic Space (Y)