Brain and CSF (F)

CSF Formation is Constant at 0.43 ml/min

Constant at $Q_{CS}$

Veins and Saggital Sinus [7.82 mmHg]

Brain and CSF (F) [11.00 mmHg]

75% of CSF formation

Transverse Sinuses and Jugular Veins (V)

[5.4 mmHg]

Brain and CSF (F)

Arteries and Capillaries (C)

Thoracic Space (Y)

Formation

25% of CSF formation

FSC

FVC

C

CF

FYC

Transverse Sinuses

and

Jugular Veins (V)

\[ 5.4 \text{ mmHg} \]

\[
\frac{dV_F}{dt} = \bar{Q}_{CF} - Z_{FS}(P_F - P_S) - Z_{FV}(P_F - P_V) \quad (1)
\]

\[
\frac{dV_S}{dt} = \bar{Q}_{CS} + Z_{FS}(P_F - P_S) - Z_{SV}(P_S - P_V) \quad (2)
\]

\[
C_{CF} \frac{dP_{FC}}{dt} + C_{FS} \frac{dP_{FS}}{dt} + C_{FV} \frac{dP_{FV}}{dt} + C_{FY} \frac{dP_{FY}}{dt} = (Q_{CF} + Q_{inf}) - (Z_{FS}P_{FS} + Z_{FV}P_{FV}) \quad (3)
\]

\[
C_{FS} \frac{dP_{FS}}{dt} = (Q_{CS} + Z_{FS}P_{FS}) - Z_{SV}P_{SV}, \quad (4)
\]

Now let $P_V$ and $P_C$ be held constant at $\bar{P}_V$ and $\bar{P}_C$ respectively, and $Q_{inf} = 0$.

\[
\begin{pmatrix}
  C_{FS} + C_F \\
  -C_F
\end{pmatrix}
\begin{pmatrix}
  \dot{P}_F \\
  \dot{P}_S
\end{pmatrix} =
\begin{pmatrix}
  -(Z_{FS} + Z_{FV}) \\
  Z_{FS}
\end{pmatrix}
\begin{pmatrix}
  P_F \\
  P_S
\end{pmatrix} + \begin{pmatrix}
  \bar{Q}_{CF} + Z_{FV}\bar{P}_V \\
  \bar{Q}_{CS} + Z_{SV}\bar{P}_V
\end{pmatrix} \quad (5)
\]

where $C_F = C_{FV} + C_{FC} + C_{FY}$.

Now let $P_F = \bar{P}_F + x(t)$ and $P_S = \bar{P}_S + y(t)$. Show that $x(t)$ and $y(t)$ satisfy

\[
\begin{pmatrix}
  C_{FS} + C_F \\
  -C_F
\end{pmatrix}
\begin{pmatrix}
  \dot{x} \\
  \dot{y}
\end{pmatrix} =
\begin{pmatrix}
  -(Z_{FS} + Z_{FV}) \\
  Z_{FS}
\end{pmatrix}
\begin{pmatrix}
  x \\
  y
\end{pmatrix} \quad (6)
\]

Show that (0,0) is the lone steady-state and it is a stable node. MAPLE.
Resistance: $Q = \frac{\Delta P}{R}$ \hspace{1cm} (R is the resistance = inverse of fluidity).

In order to accommodate the existence of a collapsed sinus and the resistance dependence upon downstream transmural pressure we consider a Starling-like resistor of the form

$$ R_{SV} = \begin{cases} 
q R_{SV} & \text{if } P_{FV} > P_{FV}^{\text{max}} = \frac{q-1}{m} + \bar{P}_{FV} \\
\left[1 + m(P_{FV} - \bar{P}_{FV})\right] R_{SV} & \text{if } P_{FV}^{\text{min}} \leq P_{FV} \leq P_{FV}^{\text{max}} \\
p R_{SV} & \text{if } P_{FV} < P_{FV}^{\text{min}} = \bar{P}_{FV} - \frac{1-p}{m}.
\end{cases} \hspace{1cm} (7) $$

Here, $q \geq 1$ and $0 \leq p \leq 1$. An example of the graph of such a resistor is given in figure 1.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{The downstream Starling-like resistor described by equation 7. Here, $x = (P_F - \bar{P}_F)$ and $z = (P_V - \bar{P}_V)$}
\end{figure}

Let

$$ x = P_F - \bar{P}_F \hspace{1cm} (8) $$

$$ y = P_S - \bar{P}_S \hspace{1cm} (9) $$

$$ z = P_V - \bar{P}_V \hspace{1cm} (10) $$

Now express $R_{SV}$ in terms of $x$ and $y$. Plug this into equation 6 and investigate the dynamics using maple.
Here are the equations in \( x \) and \( y \).

\[
(C_F + C_{FS}) \dot{x} - C_{FS} \dot{y} = (Q_{CF} + Q_{inf}) - \left( \frac{(x - y) + P_{FS}}{R_{FS}} + \frac{(x - z) + P_{FV}}{R_{FV}} \right)
\]

(11)

\[
-C_{FS} \dot{x} + C_{FS} \dot{y} = Q_{CS} + \frac{(x - y) + P_{FS}}{R_{FS}} - \frac{(y - z) + P_{SV}}{R_{SV}}
\]

(12)

\[
R_{SV} = \begin{cases} 
q \bar{R}_{SV} & \text{if } x > z + \frac{q-1}{m} \\
(1 + m(x - z)) \bar{R}_{SV} & \text{if } z - \frac{1-p}{m} \leq x \leq z + \frac{q-1}{m} \\
p \bar{R}_{SV} & \text{if } x < z - \frac{1-p}{m}
\end{cases}
\]

(13)

Start with \( q = .2 \) and \( p = 10 \).

\[
\bar{Q}_{CF} = .43 \text{ ml/min}
\]

(14)

\[
\bar{Q}_{CS} = (0.15)(6900) \text{ ml/min}
\]

(15)

\[
\bar{Q}_{FS} = 0.75 \bar{Q}_{CF} \text{ ml/min}
\]

(16)

\[
\bar{Q}_{FV} = 0.25 \bar{Q}_{CF} \text{ ml/min}
\]

(17)

\[
\bar{Q}_{SV} = \bar{Q}_{CS} + \bar{Q}_{FS} \text{ ml/min}
\]

(18)

\[
\bar{P}_F = 11 \text{ mmHg}
\]

(19)

\[
\bar{P}_S = 7.82 \text{ mmHg}
\]

(20)

\[
\bar{P}_V = 5.4 \text{ mmHg}
\]

(21)

\[
\bar{R}_{IJ} = \frac{\bar{P}_I - \bar{P}_J}{\bar{Q}_{IJ}}
\]

(22)

Start with \( q = .2 \) and \( p = 10 \). We will vary \( m \) and see how the dynamics change for various initial conditions.

<table>
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<th>( m )</th>
<th>( x(0) )</th>
<th>( y(0) )</th>
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<td>.495</td>
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</tbody>
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Brain and CSF (F)

CSF Formation is Constant at 0.43 ml/min

0.15 * 6900 ml/min

Constant at Q CS

Q FS

75\% of CSF formation

Veins and Saggital Sinus [7.82 mmHg]

Q SV

Transverse Sinuses and Jugular Veins (V) [5.4 mmHg]

Q FY

25\% of CSF formation

Brain and CSF (F) [11.00 mmHg]

Q CF

CSF Formation is Constant at 0.43 ml/min

Arteries and Capillaries (C)

Q inf = CSF infusion

C CF

Thoracic Space (Y)

C FY