1. A generalization of the undamped pendulum equation is

\[ \frac{d^2u}{dt^2} + g(u) = 0, \]  

where \( g(0) = 0 \), \( g(u) > 0 \) for \( 0 < u < k \), and \( g(u) < 0 \) for \( -k < u < 0 \); that is, \( ug(u) > 0 \) for \( u \neq 0 \), \( -k < 0 < k \). Notice that \( g(u) = \sin u \) has this property on \((-\pi/2, \pi/2)\) and corresponds to the undamped pendulum problem studied in class.

(a) Letting \( x = u \) and \( y = du/dt \), write equation (1) as a system of two equations, and show that \( x = 0, y = 0 \) is a critical point.

(b) Show that

\[ V(x, y) = \frac{1}{2}y^2 + \int_0^x g(s) \, ds, \quad -k < x < k \]

is positive definite, and use this result to show that the critical point \((0, 0)\) is stable.

2. The van der Pol equation

\[ u'' - \gamma(1-u^2)u' + u = 0 \]

where \( \gamma \) is a positive constant, describes the current \( u \) in a triode oscillator.

(a) Express the van der Pol equation as a system of first order equations.

(b) Show that \((0,0)\) is the only critical point of the system and is either an unstable spiral or unstable node.

(c) Use the first theorem of limit cycles to show that if a periodic solution exists, it must enclose \((0,0)\).

(d) Use the second theorem of limit cycles to show that a periodic solution is not contained entirely in \( D = \{(x, y) : |x| < 1 \text{ and } -\infty < y < \infty\} \).

(e) By observing Maple graphs of \( u(t) \), describe how the period \( T \) of the limit cycle depends on the value of \( \gamma \). I.e. Does the period of the limit cycle (periodic solution) increase or decrease with respect to \( \gamma \).