Homework #8 Due: Wednesday of Final's week

MTHBD/CMPBD 424 *(70 pts)*

Absolutely No Collaboration Permitted

1. May I take your order?

   For x_i, $0 \leq i \leq m$, I generated y values by $y_i = P_N(x_i) + \epsilon_i$. Here,

$$P_N(x) = a_N\, x^N + a_{N-1}\, x^{N-1} + \ldots a_1\, x + a_0 = \sum_{j=0}^{N} a_j x^j$$

   and ϵ_i is small and randomly generated from and even distribution about zero.

   You have to figure out what the coefficients of this polynomial are and translate this into a word based on the algorithm that 1 = a, 2 = b, ... 26 = z. Your word starts with the letter corresponding to a_N. There may be some filler letters at the end. These are not part of the message. Your decoded message should be an answer to the question above.

   Each of you will work on a different set of data. Go to the class website and download your data in the form: **yourlastname.mat**. In this file is a 2 x 301 matrix called data. The first row contains x-values, the second row y-values. Do not try to open this file in MATLAB. Instead the matrix is input by typing **load(yourlastname.mat)**. After you have done this a variable called **data** is now in your work space. You can get the x and y values with something along the lines of **xdata = data(1,:)** and **ydata = data(2,:)**.

   You must obtain the least squares polynomial for this data and hence decode your message. It is important that you keep increasing the degree of the least squares polynomial until the ratio of the respective variances $(\sigma^2_{N+1}/\sigma^2_N)$ is greater than 0.999. When this occurs you can assume that the degree of the polynomial is N. Recall that variance is defined by

$$\sigma_n^2 = \frac{\sum_{i=0}^{m} \left[P_n(x_i) - y_i\right]^2}{m - n}.$$

   (a) *(20 pts)* With respect to the orthogonal basis we developed in class $[q_0(x), q_1(x), \ldots q_N(x)]$ the polynomial sought can be expressed as

$$P_N(x) = c_0\; q_0(x) + c_1\; q_1(x) + \ldots c_N\; q_N(x).$$

   List these coefficients $c_0, c_1, \ldots c_N$. Please round to the nearest integer for easy viewing.

   (b) (*20 pts*) With respect to the standard basis the polynomial sought can be expressed as

$$P_N(x) = a_0 + a_1\, x + \ldots + a_N\, x^N.$$

   Your word is: $a_N\; a_{N-1}\; \ldots\; a_1\; a_0$.

   - List these coefficients rounded to the nearest integer.
   - What is your word?

   You may obtain these coefficients by converting back to the regular basis from your answer to part (a). It may be easier to obtain these coefficients by using the normal equations in the standard basis, using Matlab's **polyfit**, or using the pseudo-inverse method described at the beginning of the semester. So you can obtain this answer before completing part (a). I suggest you do so because converting from the orthogonal basis to the standard basis is tricky.

2. Given $m+1$ values of x: $\{x_i\}_{i=0}^{m}$, prove that the inner product defined by

$$\langle f, g\rangle = \sum_{i=0}^{m} f(x_i)g(x_i)$$

satisfies the defining properties of an inner product.

This requires showing that this operation satisfies:

- $\langle f, g\rangle = \langle g, f\rangle$
- $\langle f, f\rangle \geq 0$
- $\langle f, f\rangle = 0$ if and only if $f(x_i) = 0$ for all i.
- $\langle af, g\rangle = a\langle f, g\rangle$ for any real number a.
- $\langle f, g+h\rangle = \langle f, g\rangle + \langle f, h\rangle$

3. Complete the proof that the basis we developed satisfies

$$\langle q_i, q_j\rangle = 0 \quad \text{when} \quad i \neq j.$$

In other words prove that our basis is indeed orthogonal. You may use the results we obtained in class:

$$\langle q_1, q_0\rangle = 0, \quad \langle q_2, q_1\rangle = 0$$

You should prove that $\langle q_2, q_0\rangle = 0$ and then prove using induction that

$\langle q_i, q_j\rangle = 0 \quad \text{for all} \quad i \neq j.$

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- The coefficients with respect to the orthogonal basis functions are:
  (round to integers and label).

- The coefficients with respect to the standard polynomial basis functions are:
  (round to integers and label).

- May I take your order?

Hand In:

- **page 1:** This page with the answers to the above questions
- **pages $>$ 1:** The proofs requested in numbers 2 and 3 (in that order).