

Homework #7 Due: Monday April 12

(50 pts)

MTHBD/CMPBD 424

Taken from the text: page 623; Computer Problem 9.1.

Consider the heat conduction problem over the interval $x \in [0, 1]$.

$$u_{xx} = u_t \quad (1)$$

with initial condition

$$u(x, 0) = \sin(\pi x) \quad (2)$$

and the boundary conditions

$$u(0, t) = 0 \quad (3)$$

$$u(1, t) = 0 \quad (4)$$

We will numerically solve this partial differential equation in various ways and look at the error at the last time step. Let v represent the numerical approximation with space step size $= h$, time step size $= k$, and the number of time steps $= M$.

The exact solution is $u(x, t) = e^{-\pi^2 t} \sin(\pi x)$.

1. Use the **explicit** method with $h = .1$, $k = 0.005125$, and $M = 200$.

Plot the error $= (v - exact)$ at time $t = M \cdot k = 1.025$.

The max error at this time should be $\mathcal{O}(10^{-6})$.

2. Use the **explicit** method with $h = .1$, $k = 0.006$, and $M = 171$.

Plot the error $= (v - exact)$ at time $t = M \cdot k = 1.026$.

The max error at this time should be $\mathcal{O}(10^5)$.

3. In class we showed that the explicit method is **stable** if and only if

$$\frac{k}{h^2} = s < \frac{1}{(1 - \cos \theta_j)} \quad \text{where} \quad \theta_j = \frac{j\pi}{n+1} \quad \text{for} \quad 1 \leq j \leq n.$$

Here, n is the number of interior x -nodes. Show that in problem (1) this constraint is not violated but in problem (2) this constraint is violated.

4. Use the **implicit** method with $h = .1$, $k = 0.006$, and $M = 171$.

Plot the error $(v - exact)$ at time $t = M \cdot k = 1.026$.

This should resolve the large error from part (2). Why is this?

5. **Bonus** Turn in a graph of $v(x, t)$ for $x \in [0, 1]$, $t \in [0, Mk]$ for problem 1. You'll have to read about **mesh** or **surf** for plotting this.

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Here, n is the number of interior x -nodes. Show that in problem (1) this constraint is not violated but in problem (2) this constraint is violated.

Hand In:

- **page 1:** This page with the answer to problem 3.
- **page 2:** The error graph for #1.
- **page 3:** The error graph for #2.
- **page 4:** The error graph and and answer for #4.
- **page 5:** The 3D graph for bonus problem.
- **next:** Paper copies of all code.