Homework #6 Due: Monday March 29

MTHBD/CMPBD 424

1. A cylindrical pipe has a hot fluid flowing through it. Because the pressure is very high, the walls of the pipe are thick. For such a situation, the differential equation that relates temperatures in the metal wall to radial distance is

\[ r \frac{d^2 u}{dr^2} + \frac{du}{dr} = 0, \tag{1} \]

where

\[ r = \text{radial distance from the centerline}, \]
\[ u = \text{temperature}. \]

Consider a pipe with an inner radius of 1 cm and an outer radius of 2 cm containing fluid at 540°C and an external temperature of 20°C. You are to numerically solve for the temperatures within the pipe by the finite differencing method under the two boundary conditions below.

- Boundary Conditions 1:
The inner circumference has a temperature equal to the fluid temperature and the outer radius has a temperature equal to the external temperature.

- Boundary Conditions 2:
Suppose the pipe is insulated to reduce heat loss. The insulation used has the properties such that the gradient \( du/dr \) at the outer circumference is proportional to the difference in temperatures from the outer wall to the surroundings:

\[ \frac{du}{dr} \bigg|_{r=2} = 0.083 [u(2) - 20]. \]

Hand in the following

(a) Describe the finite differencing scheme and how this leads to a system of equations. Be sure to describe how the boundary conditions are incorporated for each case.

(b) Boundary Conditions 1: Make a table of step size versus maximum error. Keep reducing the step size until you run out of memory. Put this on the same page as a graph of the exact solution and the numerical approximation using a step size that results in a maximum absolute error less than 10^{-5}. State this step size.

(c) Boundary Conditions 2: Repeat part (b) for these boundary conditions.

Analytic Solution: See homework number 5
1. Describe the finite differencing scheme and how this leads to a system of equations. Be sure to describe how the boundary conditions are incorporated for each case. (you may use more pages if necessary).

Hand In:

- **first page(s)**: This page with the answer to problem 1(a).
- **next**: The table and graph from 1(b) (name in title).
- **next**: The table and graph from 1(c) (name in title).
- **next**: Paper copies of all code.