Homework #6 Due: Monday March 29

(50 pts)

MTHBD/CMPBD 424

1. A cylindrical pipe has a hot fluid flowing through it. Because the pressure is very high, the walls of the pipe are thick. For such a situation, the differential equation that relates temperatures in the metal wall to radial distance is

$$r\,\frac{d^2u}{dr^2} + \frac{du}{dr} = 0, \tag{1}$$

where

$$\begin{aligned} r &= \text{radial distance from the centerline,} \\ u &= \text{temperature.} \end{aligned}$$

Consider a pipe with an inner radius of 1 cm and an outer radius of 2 cm containing fluid at 540^oC and an external temperature of 20^oC. You are to numerically solve for the temperatures within the pipe by the finite differencing method under the two boundary conditions below.

- Boundary Conditions 1:
  The inner circumference has a temperature equal to the fluid temperature and the outer radius has a temperature equal to the external temperature.
- Boundary Conditions 2:
  Suppose the pipe is insulated to reduce heat loss. The insulation used has the properties such that the gradient du/dr at the outer circumference is proportional to the difference in temperatures from the outer wall to the surroundings:

$$\left.\frac{du}{dr}\right|_{r=2} = 0.083\,[u(2) - 20]\,.$$

Hand in the following

(a) Describe the finite differencing scheme and how this leads to a system of equations. Be sure to describe how the boundary conditions are incorporated for each case.

(b) Boundary Conditions 1: Make a table of step size versus maximum error. Keep reducing the step size until you run out of memory. Put this on the same page as a graph of the exact solution and the numerical approximation using a step size that results in a maximum absolute error less than 10^{-5}. State this step size.

(c) Boundary Conditions 2: Repeat part (b) for these boundary conditions.

Analytic Solution: See homework number 5

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1. Describe the finite differencing scheme and how this leads to a system of equations. Be sure to describe how the boundary conditions are incorporated for each case. (you may use more pages if necessary).

Hand In:

- **first page(s) :** This page with the answer to problem 1(a).
- **next:** The table and graph from 1(b) (name in title).
- **next:** The table and graph from 1(c) (name in title).
- **next:** Paper copies of all code.