Homework #5 Due: Monday March 22

(60 pts)

MTHBD/CMPBD 424

1. In class we discussed the system of second order differential equations

$$
\begin{aligned}
u_1'' + 5u_1 = 2u_2 \ &: \quad u_1(0) = 1, \quad u_1'(0) = 0 \\
u_2'' + 2u_2 = 2u_1 \ &: \quad u_2(0) = 2, \quad u_2'(0) = 0
\end{aligned}
$$

which describes the motion of two masses attached to springs in sequence with an initial displacement of each according to the initial conditions above.

   (a) Set this up as a system of first order differential equations including the initial conditions.

   (b) Numerically solve this system for $0 \le t \le 2\pi$ using matlab's **ode45** command with a relative error tolerance of 10^{-6}. This command should look something like:

   [t,x] = ode45('odefun',[0,2*pi],xo,odeset('RelTol',1e-6)).

   Plot the numerical approximations to u_1 and u_2 over the time interval $t \in [0, 2\pi]$ on the same graph. **Put your name in the title of the graph,** and clearly label which curve represent u_1 and which u_2.

2. A cylindrical pipe has a hot fluid flowing through it. Because the pressure is very high, the walls of the pipe are thick. For such a situation, the differential equation that relates temperatures in the metal wall to radial distance is

$$r\,\frac{d^2u}{dr^2} + \frac{du}{dr} = 0, \tag{1}$$

where

$$\begin{aligned} r &= \text{radial distance from the centerline,} \\ u &= \text{temperature.} \end{aligned}$$

Consider a pipe with an inner radius of 1 cm and an outer radius of 2 cm containing fluid at 540^{o}C and an external temperature of 20^{o}C. You are to numerically solve for the temperatures within the pipe by the shooting method under the two boundary conditions below.

- Boundary Conditions 1:

  The inner circumference has a temperature equal to the fluid temperature and the outer radius has a temperature equal to the external temperature.

- Boundary Conditions 2:

  Suppose the pipe is insulated to reduce heat loss. The insulation used has the properties such that the gradient du/dr at the outer circumference is proportional to the difference in temperatures from the outer wall to the surroundings:

$$\left.\frac{du}{dr}\right|_{r=2} = 0.083\,[u(2) - 20]\,.$$

Hand in the following for each type of boundary condition (4 graphs total) each with your name in the title.

(a) Hand in the graphs of your first three shots on the same axes. You may perform the shots however you like. Label them accordingly. State the associated guesses at $u'(1)$ for each solution.

(b) Hand in a plot of the exact solution and your third shot on the same axes and state the maximum absolute error between the approximation and the exact solution over the interval $r \in [1, 2]$.

Analytic Solution

The differential equation

$$r\,\frac{d^2u}{dr^2} + \frac{du}{dr} = 0,$$

has an exact solution of the form

$$u(r) = C_1 \ln(r) + C_2.$$

C_1 and C_2 may be determined from the boundary conditions.

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1. Describe the system of first order differential equations obtained from the system of second order differential equations in problem 1. Be sure to include the initial conditions.

Hand In:

- **Page 1:** This page with the answer to problem 1(a).
- **Page 2:** The graphs of u_1 and u_2 from problem 1(b) (name in title).
- **Page 3:** The graph from problem 2(a) Boundary Conditions 1 (name in title).
- **Page 4:** The graph from problem 2(b) Boundary Conditions 1 (name in title).
- **Page 5:** The graph from problem 2(a) Boundary Conditions 2 (name in title).
- **Page 6:** The graph from problem 2(b) Boundary Conditions 2 (name in title)