Homework #2 Due: Friday February 13

(60 pts)

MTHBD/CMPBD 424

1. Consider the matrix A defined by

$$A = \begin{pmatrix} 3 & 1 & -4 & 3 \\ 1 & 8 & -3 & 5 \\ 1 & 2 & 4 & 4 \\ 1 & 2 & 0 & 8 \end{pmatrix}$$

We are going to try to find the largest and smallest eigenvalues of this matrix using the power method. For these methods use the $||\cdot||_\infty$ norm in your algorithm.

   (a) Use the built in matlab function **eig** to find the eigenvalues and eigenvectors of A. Type **help eig** or look it up in the help index to find the appropriate usage. Put your results on the cover page attached.

   (b) Use the power method to find the largest eigenvalue. On the first time through find the ratio by evaluating the first term in each vector. On the second time use any other term. Give the eigenvalue and associated eigenvector in each case. Explain what went wrong on the first try.

   (c) Use the inverse power method to find the smallest eigenvalue and the associated eigenvector. Explain why your associated eigenvector is different than that given by matlab's **eig** function.

2. Consider the under-determined system $Ax = b$:

$$\begin{pmatrix} 1 & 2 & 0 \\ 1 & 2 & 1 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$$

   (a) Use matlabs's built in function **svd(A)** to find the singular value decomposition of A. Clearly state the results.

   (b) Find the minimal solution to the system using the pseudo-inverse of the matrix A. You may use the MATLAB command **pinv(A)** for this problem. This command generates the pseudo-inverse of A.

   (c) Use calculus and minimize the square of the distance from a solution to the point (0,0,0). Verify that your answer to part (b) is indeed the solution of minimum 2-norm. See me if you need help doing the calculus.

3. It is assumed by the medical community that the relationship between blood pressure in the foot and the pelvis (femoral region) is described by the equation

$$P_{foot} - P_{fem} = \overline{P} + \alpha \sin(\theta) \tag{1}$$

where θ is the angle of elevation: $\theta = 0$ is lying down, and $\theta = 90^o$ is standing upright. Here, $\overline{P}$ is a baseline pressure difference and, on the arterial side, should be negative. When standing ($\theta = 90^o$) the change in pressure difference is due to the weight of the blood column between the two locations.

We are given a table of data from arterial side[1]

θ	0	10	30	75	-10	-30	-75
P_{fem}	85.7700	89.7800	97.0200	109.6000	83.7400	76.5600	75.6200
P_{foot}	83.4714	100.2710	133.1866	177.3880	65.8014	31.1565	-4.4342

Find the parameters $\overline{P}$ and α in equation 1 that generate a curve of best fit in the least squares sense to the data presented in the table. (Be sure to convert angles to radians).

(a) Make sure to clearly describe the linear system which you are trying to "solve" in the least squares sense. Remember you are trying to solve for $\overline{P}$ and α.

(b) State the calculated values of the parameters $\overline{P}$ and α.

(c) Plot the curve with these parameters and the data points on the same graph. Does it look like a good fit?

(d) Consider the same data taken from the venous side.

θ	0	10	30	75	-10	-30	-75
P_{fem}	7.1000	7.8000	11.1000	16.0000	4.7000	2.7000	1.3000
P_{foot}	20.7500	27.0000	54.3000	89.7000	15.1000	6.5000	-2.4000

Play the same game as you did in parts (a)-(c) with this data. Only hand in the plot of the curve and data. Does it look like a good fit. Does equation (1) seem valid for blood pressures on the venous side?

Note: This data is available from the website as a MATLAB file. To avoid translation errors, I suggest you download these files instead of entering the data by hand. This file is called **FemFoot.m**. If you run it, it will load the data, calculate pressure differences and plot them against the angle of elevation. Figure 1 will be arterial data. Figure 2 will be venous data. When you generate your curve plots you will want to superimpose them onto these graphs either with the **hold on** command or by including them in a single plot command. For this problem you will again want to use the pseudo-inverse of the coefficient matrix, or if you paid close attention to the least-squares (FYI) lecture in 423 you may use that technique as well.

[1] Katkov, V.E. and Chestukhin, V.V. Blood pressure and oxygenation in different cardiovascular compartments of a normal man during postural changes (1980), *Aviat. Space Environ. Med.*, 51, 11, 1234-1242.

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1. (a) MATLAB determined eigenvalues and eigenvectors: Use only 4 decimal places for the terms in the eigenvectors.

$\lambda_1 =$ $\lambda_2 =$ $\lambda_3 =$ $\lambda_4 =$

$$\eta_1 = \begin{pmatrix} \\ \\ \\ \\ \end{pmatrix} \quad \eta_2 = \begin{pmatrix} \\ \\ \\ \\ \end{pmatrix} \quad \eta_3 = \begin{pmatrix} \\ \\ \\ \\ \end{pmatrix} \quad \eta_4 = \begin{pmatrix} \\ \\ \\ \\ \end{pmatrix}$$

Hand In:

- **Page 1:** This page with the eigenvalues and associated eigenvectors for 1(a)
- **Page 2:** The answer to problems 1(b) and 1(c).
- **Page 3:** The answer to problems 2(a) and 2(b).
- **Page 4:** The answer to problem 2(c).
- **Page 5:** The answer to problems 3(a) and 3(b).
- **Page 6:** The graph and comments for problem 3(c).
- **Page 7:** The graph and comments for problem 3(d).
- **Next pages:** Paper copies of the code used to answer numbers 1 and 3 in that order.