Homework #7 Due: Wednesday April 12

(50 pts)

MTHBD/CMPBD 424

Consider the heat conduction problem over the interval $x \in [0, 1]$.

$$u_{xx} = u_t \tag{1}$$

with initial condition

$$u(x, 0) = \sin(\pi x) \tag{2}$$

and the boundary conditions

$$u(0, t) = 0 \tag{3}$$
$$u(1, t) = 0 \tag{4}$$

We will numerically solve this partial differential equation in various ways and look at the error at the last time step. Let v represent the numerical approximation with space step size $= h$, time step size $= k$, and the number of time steps $= M$.

The exact solution is $u(x, t) = e^{-\pi^2 t} \sin(\pi x)$.

1. Use the **explicit** method with $h = .1$, $k = 0.005125$, and $M = 200$.

   Plot the relative error $= \frac{v(t)-u(t)}{u(t)}$ at time $t = M \cdot k = 1.025$.
   If $v = u = 0$ set the relative error $= 0$.

   The max |relative error| at this time is between 0.10 and 0.20.

2. Use the **explicit** method with $h = .1$, $k = 0.006$, and $M = 171$.

   Plot the relative error $= \frac{v(t)-u(t)}{u(t)}$ at time $t = M \cdot k = 1.026$.
   If $v = u = 0$ set the relative error $= 0$.

   The max |relative error| at this time is $\approx 10^7$.

3. In class we showed that the explicit method is **stable** if and only if

   $$\frac{k}{h^2} = s < \frac{1}{(1 - \cos\theta_j)} \quad \text{where} \quad \theta_j = \frac{j\pi}{n+1} \quad \text{for} \quad 1 \le j \le n.$$

   Here, n is the number of interior x-nodes. Show that in problem (1) this constraint is not violated but in problem (2) this constraint is violated.

4. Use the **implicit** method with $h = .1$, $k = 0.006$, and $M = 171$.

   Plot the relative error $= \frac{v(t)-u(t)}{u(t)}$ at time $t = M \cdot k = 1.026$.
   If $v = u = 0$ set the relative error $= 0$.

   This should resolve the large error from part (2). Why is this?

5. **Bonus** Turn in a graph of $v(x, t)$ for $x \in [0, 1]$, $t \in [0, Mk]$ for problem 1. You'll have to read about **mesh** or **surf** for plotting this.

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  Here, n is the number of interior x-nodes. Show that in problem (1) this constraint is not violated but in problem (2) this constraint is violated.

Hand In:

- **page 1:** This page with the answer to problem 3.
- **page 2:** The error graph for #1. (put code in my P-drive)
- **page 3:** The error graph for #2. (put code in my P-drive)
- **page 4:** The error graph and and explanation for #4. (put code in my P-drive)
- **page 5:** The 3D graph for bonus problem. (put code in my p-drive)