

Homework #6 Due: Wednesday March 29

(50 pts)

MTHBD/CMPBD 424

1. A cylindrical pipe has a hot fluid flowing through it. Because the pressure is very high, the walls of the pipe are thick. For such a situation, the differential equation that relates temperatures in the metal wall to radial distance is

$$r \frac{d^2u}{dr^2} + \frac{du}{dr} = 0, \quad (1)$$

where

$$\begin{aligned} r &= \text{radial distance from the centerline,} \\ u &= \text{temperature.} \end{aligned}$$

Consider a pipe with an inner radius of 1 cm and an outer radius of 2 cm containing fluid at 540°C and an external temperature of 20°C. You are to numerically solve for the temperatures within the pipe by the finite differencing method under the two boundary conditions below.

- Boundary Conditions 1:

The inner circumference has a temperature equal to the fluid temperature and the outer radius has a temperature equal to the external temperature.

- Boundary Conditions 2:

Suppose the pipe is insulated to reduce heat loss. The insulation used has the properties such that the gradient du/dr at the outer circumference is proportional to the difference in temperatures from the outer wall to the surroundings:

$$\left. \frac{du}{dr} \right|_{r=2} = -0.083 [u(2) - 20].$$

Hand in the following

- (a) Describe the finite differencing scheme and how this leads to a system of equations. Be sure to describe how the boundary conditions are incorporated for each case.
- (b) Boundary Conditions 1: Make a table of step size versus maximum error. Keep reducing the step size until you run out of memory. Put this on the same page as a graph of the exact solution and the numerical approximation using a step size that results in a maximum absolute error less than 10^{-5} . State this step size. Your table should look something like:

h	Max Error
.1	
.01	
.001	
⋮	

- (c) Boundary Conditions 2: Repeat part (b) for these boundary conditions.

Analytic Solution: See homework number 5

1. Describe the finite differencing scheme and how this leads to a system of equations. Be sure to describe how the boundary conditions are incorporated for each case. (you may use more pages if necessary).

Hand In:

- **first page(s)** : This page with the answer to problem 1(a).
- **next**: The table and graph from 1(b) (well-labelled).
- **next**: The table and graph from 1(c) (well-labelled).
- **next**: The code for both problems should be dropped in my P-drive.