1. A cylindrical pipe has a hot fluid flowing through it. Because the pressure is very high, the walls of the pipe are thick. For such a situation, the differential equation that relates temperatures in the metal wall to radial distance is

\[ r \frac{d^2 u}{dr^2} + \frac{du}{dr} = 0, \]  

where

\( r \) = radial distance from the centerline,
\( u \) = temperature.

Consider a pipe with an inner radius of 1 cm and an outer radius of 2 cm containing fluid at \( 540^\circ C \) and an external temperature of \( 20^\circ C \). You are to numerically solve for the temperatures within the pipe by the finite differencing method under the two boundary conditions below.

- **Boundary Conditions 1:**
  The inner circumference has a temperature equal to the fluid temperature and the outer radius has a temperature equal to the external temperature.

- **Boundary Conditions 2:**
  Suppose the pipe is insulated to reduce heat loss. The insulation used has the properties such that the gradient \( du/dr \) at the outer circumference is proportional to the difference in temperatures from the outer wall to the surroundings:

\[ \left. \frac{du}{dr} \right|_{r=2} = -0.083 [u(2) - 20]. \]

**Hand in the following**

(a) Describe the finite differencing scheme and how this leads to a system of equations. Be sure to describe how the boundary conditions are incorporated for each case.

(b) **Boundary Conditions 1:** Make a table of step size versus maximum error. Keep reducing the step size until you run out of memory. Put this on the same page as a graph of the exact solution and the numerical approximation using a step size that results in a maximum absolute error less than \( 10^{-5} \). State this step size. Your table should look something like:

<table>
<thead>
<tr>
<th>h</th>
<th>Max</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>.01</td>
<td></td>
<td></td>
</tr>
<tr>
<td>.001</td>
<td></td>
<td></td>
</tr>
<tr>
<td>:</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(c) **Boundary Conditions 2:** Repeat part (b) for these boundary conditions.

**Analytic Solution:** See homework number 5
1. Describe the finite differencing scheme and how this leads to a system of equations. Be sure to describe how the boundary conditions are incorporated for each case. (you may use more pages if necessary).

Hand In:

- **first page(s)**: This page with the answer to problem 1(a).
- **next**: The table and graph from 1(b) (well-labelled).
- **next**: The table and graph from 1(c) (well-labelled).
- **next**: The code for both problems should be dropped in my P-drive.