1. Interpolating data with three forms of polynomials. (10 pts)

You are asked to find the coefficients of the 4th degree polynomial which interpolates the following data:

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>15</td>
<td>-17</td>
<td>-125</td>
<td>-285</td>
<td>-377</td>
</tr>
</tbody>
</table>

- Vandermonde Method: Find the coefficients \((a_0, a_1, \ldots, a_n)\) of the interpolating polynomial in the form:
  \[ p_n(x) = a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n \]

Find these coefficients using the Vandermonde matrix.

- Lagrange interpolation polynomial: Find the coefficients \((b_0, b_1, \ldots, b_n)\) of the interpolating polynomial in the form:
  \[ p_n(x) = b_0 \ell_0(x) + b_1 \ell_1(x) + \ldots + b_n \ell_n(x) \]
  where \( \ell_i(x) = \prod_{\substack{j=0 \atop j \neq i}}^{n} \frac{x - x_j}{x_i - x_j} \)

2. For the interpolation nodes (x-values) -1, 1, 3, 4 plot the Lagrange basis functions \(l_i(x)\) for \(0 \leq i \leq 3\) on the same axes. Describe the key features of these curves that are critical for their use in interpolating data points.