Math 250, Section 1, TEST 1

Graphing calculators are allowed during this exam, except TI-89’s and TI-92’s. All non-trivial answers must be justified by relevant work. In these non-trivial cases, a correct answer without any work will receive no credit.

1. Solve the following initial value problems. (30 pts)

(a) \( y' - 2y = e^{2t}, \quad y(0) = 2 \)

(b) \( y' - y^2 \sin(x) = 0, \quad y(0) = \frac{1}{2} \)
2. Solve the following differential equations. You need not solve y explicitly in terms of x. (25 pts)

(a) \( \frac{dy}{dx} = \frac{x - e^{-x}}{y + e^y} \)

(b) \( (3x^2 - 2xy + 2) + (6y^2 - x^2 + 3) \frac{dy}{dx} = 0 \)
3. A tank initially contains 100 liters of pure water. A mixture containing a concentration of $\gamma$ g/liter of salt enters the tank at a rate of 2 liters/min, and the well-stirred mixture leaves the tank at the same rate. Answer the following questions. Keep in mind that the answers will depend on the unknown parameter $\gamma$. (25 pts)

(a) Let $Q(t)$ denote the grams of salt in the tank at any time $t$. Derive a differential equation which has $Q(t)$ as the solution. **Include an initial value.**

(b) Solve the initial value problem derived in part (a).

(c) What is the limiting value of $Q$ as $t \to \infty$?
4. Determine, without solving the problem, an interval in which the solution of the given differential equation is certain to exist. 

\[(t - 3)y' + \sqrt{t} y = e^{2t}, \quad y(1) = 2\]

Your answer:

5. Consider the differential equation \( \frac{dy}{dt} = (y - 1)(3 - y). \) (10 pts)

(a) Make a rough sketch of the graph of \( y' \) versus \( y. \)

(b) Suppose \( y(0) = 2. \) Without solving the differential equation, determine what happens to \( y \) as \( t \) gets large. Explain your reasoning.

(c) Suppose \( y(0) = 4. \) Without solving the differential equation, determine what happens to \( y \) as \( t \) gets large. Explain your reasoning.