Homework assignment for Section 10.7 in Boyce and Diprima

1. Find the solution to the following wave equations. Where \( a^2 = 4, L = 10, \)

\[
f(x) = \begin{cases} 
  x & 0 \leq x \leq 5 \\
  10-x & 5 \leq x \leq 10 
\end{cases}
\]

and

\[g(x) = \sin \left( \frac{\pi x}{10} \right)\]

Find the form of the coefficients involved in the solution, you need not carry out the integration.

(a) \( a^2u_{xx} = u_{tt} \)

\[u(0,t) = 0, \quad u(L,t) = 0 \]

\[u(x,0) = f(x), \quad u_t(x,0) = 0 \]

(b) \( a^2u_{xx} = u_{tt} \)

\[u(0,t) = 0, \quad u(L,t) = 0 \]

\[u(x,0) = 0, \quad u_t(x,0) = g(x) \]

(c) \( a^2u_{xx} = u_{tt} \)

\[u(0,t) = 0, \quad u(L,t) = 0 \]

\[u(x,0) = f(x), \quad u_t(x,0) = g(x) \]

Find the solution to this last equation in terms of the solutions to the first two.

2. Number 13 Section 10.7

3. Number 14 Section 10.7

4. Find the solution to problem 1(a) in terms of the original displacement function. Here you should be able to define the solution explicitly.