In each of the following problems, find the steady-state solution of the heat equation \( \alpha^2 u_{xx} = u_t \) that satisfies the given set of boundary conditions.

1. \( u(0,t) = 10, \quad u(50,t) = 40 \)
2. \( u_x(0,t) = 0, \quad u(L,t) = 0 \)
3. \( u(0,t) = 0, \quad u_x(L,t) = 0 \)
4. \( u_x(0,t) - u(0,t) = 0, \quad u(L,t) = T \)

In each of the following boundary value problems, find

(a) The steady state solution.
(b) The boundary value problem that determines the transient solution.
(c) The form of the transient solution. You need not solve for the Fourier coefficients explicitly.
(d) The solution to the original boundary value problem in terms of the transient and steady state solutions.

5. \( u_{xx} = u_t, \quad 0 < x < 30, \quad t > 0 \)
   \( u(0,t) = 20, \quad u(30,t) = 50 \)
   \( u(x,0) = 60 - 2x, \quad 0 < x < 30 \)

6. \( \alpha^2 u_{xx} = u_t, \quad 0 < x < 20, \quad t > 0 \)
   \( u(0,t) = 0, \quad u(20,t) = 60 \)
   \( u(x,0) = 25, \quad 0 < x < 20 \)

Find the form of the solution to the following heat conduction problems where the boundary conditions imply insulated ends. You need not solve for the Fourier coefficients explicitly.

7. \( u_{xx} = u_t, \quad 0 < x < 30, \quad t > 0 \)
   \( u_x(0,t) = 0, \quad u_x(30,t) = 0 \)
   \( u(x,0) = 60 - 2x, \quad 0 < x < 30 \)

8. \( \alpha^2 u_{xx} = u_t, \quad 0 < x < 20, \quad t > 0 \)
   \( u_x(0,t) = 0, \quad u_x(20,t) = 0 \)
   \( u(x,0) = 25, \quad 0 < x < 20 \)