1. Consider a smooth curve $C$ represented by the vector function $\mathbf{r}(t)$.

(a) **T**  **F**  The unit normal vector is always orthogonal to the velocity vector.

(b) **T**  **F**  The acceleration vector is always orthogonal to the unit tangent vector.

(c) **T**  **F**  $D_t[r(t) \cdot r(t)] = 2 \mathbf{r}'(t) \cdot \mathbf{r}(t)$

(d) **T**  **F**  If $||\mathbf{v}(t)||$ is constant then $\mathbf{a}(t)$ is orthogonal to $\mathbf{v}(t)$.

(e) **T**  **F**  If $\mathbf{r}(t)$ represents position and $s(t)$ represents the associated arc length, then $\frac{ds}{dt} = ||\mathbf{r}'(t)||$.

(f) **T**  **F**  If the curvature at a point $P$ is 4, then the radius of curvature is 0.25.

(g) **T**  **F**  The acceleration vector is always perpendicular to the unit tangent vector.

(h) **T**  **F**  The acceleration vector lies in the same plane as the unit tangent vector and the unit normal vector.

(i) **T**  **F**  The velocity vector is parallel to the unit tangent vector.

(j) **T**  **F**  If $\mathbf{r}(t)$ represents position and $s(t)$ represents the associated arc length, then $\frac{ds}{dt} = \mathbf{r}'(t)$.

2. The principal unit normal vector to the graph of $y = \cos x$ at the point $(\pi, -1)$ is

(a) $<1, 0>$  (b) $<0, -1>$  (c) $<-1, 0>$  (d) $<0, 1>$

3. Consider a curve in space generated by $\mathbf{r}(t) = (2t + 1) \mathbf{i} - 3t \mathbf{j} + 10 \mathbf{k}$.

The length of this curve as $t$ goes from 0 to 2 is

(a) $\sqrt{13}$  (b) $4\sqrt{13}$  (c) $2\sqrt{5}$  (d) $2\sqrt{13}$  (e) $\sqrt{5}$

4. The position vector of a particle at time $t$ is given by $\mathbf{r}(t) = t \mathbf{i} + t^2 \mathbf{j} + 2t \mathbf{k}$

The speed of the particle at time $t = 1$ is

(a) $\sqrt{13}$  (b) 3  (c) $\sqrt{10}$  (d) $2\sqrt{10}$  (e) 5

5. A baseball is hit 3 feet above the ground at 128 feet per second and at an angle of $\frac{\pi}{6}$ with respect to the ground. Assume that the only force acting on the ball after it is hit is that due to gravity. ($g = 32 \text{ feet}/(\text{sec})^2$). What is the height of the ball at $t = 2$ seconds? Answers are in feet.

(a) 67  (b) 62  (c) 32  (d) 48  (e) 128

6. The curve generated by $\mathbf{r}(t) = t \mathbf{i} + t^2 \mathbf{j}$ has the same shape as the curve

(a) $x = y^2$  (b) $y = x^2$  (c) $y = \sqrt{x}$  (d) $y = 2x$  (e) $y = x$

7. Find $\mathbf{r}(t)$ given $\mathbf{r}'(t) = e^{-t} \mathbf{i} - t^{1/2} \mathbf{j} + \mathbf{k}$ and $\mathbf{r}(0) = -\mathbf{i} + \mathbf{k}$
8. The position vector of a particle at time $t$ is given by $\mathbf{r}(t) = 3t \mathbf{i} + 4\sin t \mathbf{j} + 4\cos t \mathbf{k}$

(a) Determine the velocity vector at time $t = \pi$.
(b) Determine the acceleration vector at time $t = \pi$.
(c) Determine the speed at time $t = \pi$.
(d) Determine the unit tangent vector $\mathbf{T}$ at time $t = \pi$.
(e) Determine the principal unit normal vector $\mathbf{N}$ at $t = \pi$.
(f) Determine the tangential component of acceleration at $t = \pi$.
(g) Determine the normal component of acceleration at $t = \pi$.
(h) Determine the curvature of the path at time $t = \pi$.

9. Represent the intersection of the two surfaces as a vector-valued function. You need not sketch the curve. Surface 1: $x - z^2 - y^2 = 0$ and surface 2: $z = 2y^2$

10. Find a set of parametric equations for the line tangent to the space curve generated by

$$\mathbf{r}(t) = < e^{-t}, \sin t, 4t >$$

at $t = 0$.

11. Find the open interval(s) on which the curve given by $\mathbf{r}(t)$ is smooth.

$$\mathbf{r}(t) = t^2 \mathbf{i} + (e^t - t) \mathbf{j} + \mathbf{k}$$

12. Evaluate the definite integral $\int_{0}^{\pi/2} [2 \cos t \mathbf{i} + 2 \sin t \mathbf{j} + \mathbf{k}] \, dt$

13. Represent the plane curve $y = x^2 + 2$ by a vector valued function $\mathbf{r}(t)$.

14. Sketch the graph of the vector valued function $\mathbf{r}(t) = \sin(t)\mathbf{i} - 2\cos(t)\mathbf{j}$ for $0 \leq t \leq \frac{3\pi}{2}$.

15. A baseball is hit 3 feet above the ground at 128 feet per second and at an angle of $\frac{\pi}{6}$ with respect to the ground. Assume that the only force acting on the ball after it is hit is that due to gravity. ($g = 32$ feet/(sec)$^2$).

(a) Use the fact that acceleration is constant and given by $\mathbf{a}(t) = -g \mathbf{j}$ to derive a function for the position of the ball $\mathbf{r}(t)$ for any time $t$. Show your derivation below and write the result in the box.

(b) What is the maximum height the ball reaches?

16. The DNA molecule has the shape of a double helix. The radius of each helix is about 10 angstroms (1 angstrom = $10^{-8}$ cm). Each helix rises about 34 angstroms during each complete turn and there are about $2.9 \times 10^8$ complete turns, so the vector valued function defining each helix is

$$\mathbf{r}(t) = (10 \cos t)\mathbf{i} + (10 \sin t)\mathbf{j} + \left(\frac{34}{2\pi}t\right)\mathbf{k}, \quad 0 \leq t \leq 2\pi \times 10^8.$$