Sample Exam 1

The actual exam will not be this long and WILL include some questions not represented here.

1. True-False
   (a) **T** **F** The vectors $\langle 2, -1, 3 \rangle$ and $\langle 4, -2, 6 \rangle$ are parallel.
   (b) **T** **F** The vectors $\langle 1, 2, 3 \rangle$ and $\langle 1, -5, 3 \rangle$ are orthogonal.
   (c) **T** **F** The vectors $\langle 4, 1, 2 \rangle$ and $\langle -2, 4, 7 \rangle$ have the same length.
   (d) **T** **F** The vector $\langle 1, 1, 0 \rangle$ is a unit vector.
   (e) **T** **F** The vector $\langle 2, 1, -1 \rangle$ is normal (perpendicular) to the plane $2x + y - z = 3$.
   (f) **T** **F** If $w = u \times v$, then $w$ is orthogonal to both $u$ and $v$.
   (g) **T** **F** If $w = 2u$, then $||u \times v|| = 2 ||w \times v||$.

2. If $u$ has length 3, $v$ has length 2, and the angle between $u$ and $v$ is 60°, then $u \cdot v =$
   (a) $1/2$  (b) $6$  (c) $3$  (d) $3/2$  (e) $3\sqrt{3}$

3. If $u = 2i + 2j$ and $v = i + j + k$, then $\text{proj}_v u$ (the projection of $u$ onto $v$) is
   (a) $\frac{4}{3} u$  (b) $\frac{4}{3} v$  (c) $2 v$  (d) $2$  (e) $\sqrt{2} v$

4. If $u = <3, 0, 4>$ then the unit vector in the direction of $u$ is
   (a) $\frac{3}{5} 0, \frac{1}{5}$  (b) $\frac{5}{3} 0, \frac{5}{3}$  (c) $\frac{1}{2} 1, \frac{2}{3}$  (d) $5$  (e) $\frac{3}{5} 0, \frac{4}{5}$

5. The surface whose equation in cylindrical coordinates is given by $r = 3$ is
   (a) a cone  (b) a cylinder  (c) a sphere  (d) a plane  (e) an ellipsoid

6. In cylindrical coordinates the point $(r, \theta, z) = (4, \pi/6, 3)$. This point in rectangular (cartesian) coordinates is
   (a) $(\sqrt{2}/2, \sqrt{2}/2, 3)$  (b) $(2/1, 2, 3)$  (c) $(2\sqrt{3}, 2, 3)$  (d) $(2\sqrt{3}, 2, 5)$  (e) $(2\sqrt{3}, 2, 9)$

7. The point of intersection of the line given by $\frac{x + 2}{2} = \frac{y - 1}{8} = z + 2$ and the plane $y = 9$ is
   (a) $(3, 2, 1)$  (b) $(5, 9, 7)$  (c) $(1, 9, 5)$  (d) $(0, 9, -1)$  (e) $(1, 2, 5)$

8. Let $u = \sqrt{2} j - k$, $v = \sqrt{2} i + k$ and $\theta$ be the angle between these vectors. In this case, $\cos(\theta)$ is
   (a) $-1 \frac{3}{3}$  (b) $\frac{1}{\sqrt{2}}$  (c) $\frac{-1}{\sqrt{2}}$  (d) $\frac{1}{3}$  (e) $\sqrt{2}$
9. If \( u = \langle 1, -1 \rangle \) and \( v = \langle -1, 2 \rangle \) then \( ||u + v|| \) is
   (a) \( \sqrt{13} \) (b) 1 (c) \( \sqrt{5} \) (d) 3 (e) \( \sqrt{2} \)

10. A vector \( v \) has magnitude equal to 2 and makes an angle of -45° with the positive x-axis. This vector in component form is then
   (a) \( \left\langle \frac{-1}{2}, \frac{1}{2} \right\rangle \) (b) \( \langle 2, -2 \rangle \) (c) \( \langle -\sqrt{2}, \sqrt{2} \rangle \) (d) \( \langle \sqrt{2}, -\sqrt{2} \rangle \) (e) \( \left\langle \frac{1}{2}, \frac{1}{2} \right\rangle \)

11. Let \( u = \langle 1, 0, 2 \rangle \) and \( v = \langle 1, 1, 3 \rangle \). Find a unit vector that is orthogonal to both \( u \) and \( v \).

12. Let \( P(1,3,2) \), \( Q(3,1,1) \), \( R(-1,-2,3) \) be three points in \( \mathbb{R}^3 \).
   (a) Find the vector (in component form) from \( P \) to \( Q \).
   (b) Find the parametric equations for the line in space through points \( P \) and \( Q \). Parametric equations describe \( x \), \( y \), and \( z \) in terms of a parameter, usually \( t \).
   (c) Find a vector (in component form) that is orthogonal to \( \vec{PQ} \) and \( \vec{PR} \).
   (d) Find an equation for the plane determined by the points \( P,Q \), and \( R \).

13. Consider the surface defined by \( z - \frac{x^2}{4} - y^2 = 0 \).
   (a) Sketch the trace of the surface in the plane \( z = 4 \). Label the axes and clearly indicate at least two points on the trace.
   (b) Sketch the trace of the surface in the \( yz \)-plane. Label the axes and clearly indicate at least two points on the trace.
   (c) Sketch the trace of the surface in the \( xz \)-plane. Label the axes and clearly indicate at least two points on the trace.
   (d) Sketch the surface in 3-space. Label the axes.

14. Consider the vector \( u \) in the \( yz \)-plane of length 4 making an angle of 30° with the positive \( y \)-axis.
   (a) Write the vector \( u \) in standard unit vector notation (as a linear combination of \( i \), \( j \) and \( k \)).
   (b) Write the vector in component form.
   (c) Sketch the vector \( u \).

15. Consider the surface in 3-space defined by the equation \( x^2 + y^2 = 4y \).
   (a) Sketch and describe the surface in 3-space.
   (b) Convert the equation into cylindrical coordinates.

16. **Bonus** Find the point \( (x_1, y_1, z_1) \) that results when the point \( (x_0, y_0, z_0) \) is projected onto the plane \( ax + by + cz + d = 0 \).