1. Consider a smooth curve $C$ represented by the vector function $r(t)$. \(10\) pts

(a) T  F  If the curvature at a point $P$ is 4, then the radius of curvature is 0.25.

(b) T  F  The acceleration vector is always perpendicular to the unit tangent vector.

(c) T  F  The acceleration vector lies in the same plane as the unit tangent vector and the unit normal vector.

(d) T  F  The velocity vector is parallel to the unit tangent vector.

(e) T  F  If $r(t)$ represents position and $s(t)$ represents the associated arc length, then \( \frac{ds}{dt} = r'(t) \).

2. The principal unit normal vector to the graph of $y = \sin x$ at the point \((\pi/2, 1)\) is \(5\) pts

(a) \(<1, 0>\)  (b) \(<0, 1>\)  (c) \(<-1, 0>\)  (d) \(<0, -1>\)

3. Represent the plane curve $y = x^2 + 2$ by a vector valued function $r(t)$. \(5\) pts

your answer:

4. Sketch the graph of the vector valued function $r(t) = \cos(t)i + \sin(t)j$ for $0 \leq t \leq 2\pi$. \(5\) pts
5. The position vector of a particle at time \( t \) is given by

\[
\mathbf{r}(t) = \cos(2t) \, \mathbf{i} + \sin(2t) \, \mathbf{j} + t \, \mathbf{k}
\]

(a) Determine the velocity at time \( t = \pi \).

(b) Determine the speed at time \( t = \pi \).

(c) Determine the acceleration at time \( t = \pi \).

(d) Determine the unit tangent vector \( \mathbf{T} \) at time \( t = \pi \).

(e) Determine the principal unit normal vector \( \mathbf{N} \) at \( t = \pi \).

(f) Determine the tangential component of acceleration at \( t = \pi \).

(g) Determine the normal component of acceleration at \( t = \pi \).

(h) Determine the curvature of the path at time \( t = \pi \).
6. A baseball is hit 3 feet above the ground at 128 feet per second and at an angle of $\frac{\pi}{6}$ with respect to the ground. Assume that the only force acting on the ball after it is hit is that due to gravity. ($g = 32 \text{ feet/}(\text{sec})^2$).

(a) Use the fact that acceleration is constant and given by $\mathbf{a}(t) = -g \mathbf{j}$ to derive a function for the position of the ball $\mathbf{r}(t)$ for any time $t$. Show your derivation below and write the result in the box.

(b) What is the maximum height the ball reaches?

your answer:
7. The DNA molecule has the shape of a double helix. The radius of each helix is about 10 angstroms (1 angstrom $= 10^{-8}$ cm). Each helix rises about 34 angstroms during each complete turn and there are about $2.9 \times 10^8$ complete turns, so the vector valued function defining each helix is 

$$\mathbf{r}(t) = (10 \cos t) \mathbf{i} + (10 \sin t) \mathbf{j} + \left( \frac{34}{2\pi} t \right) \mathbf{k}, \quad 0 \leq t \leq 2\pi \times 10^8.$$

Determine the length of the helix.

$$\text{your answer:}$$