Summary of Vectors, Dot Products and Cross Products

- A vector is a directed line segment translated so that the initial point is at the origin. They have two different notations.
  \[ \mathbf{u} = \langle u_1, u_2, u_3 \rangle \] called component form,
  and
  \[ \mathbf{u} = u_1 \mathbf{i} + u_2 \mathbf{j} + u_3 \mathbf{k} \] where \( \mathbf{i} = \langle 1, 0, 0 \rangle \), \( \mathbf{j} = \langle 0, 1, 0 \rangle \), and \( \mathbf{k} = \langle 0, 0, 1 \rangle \)

- The vector from \( P(x_0, y_0, z_0) \) to \( Q(x_1, y_1, z_1) \) is given by
  \[ \mathbf{u} = \overrightarrow{PQ} = \langle x_1 - x_0, y_1 - y_0, z_1 - z_0 \rangle \]

- The length, magnitude, or norm of \( \mathbf{u} \) is given by
  \[ ||\mathbf{u}|| = \sqrt{u_1^2 + u_2^2 + u_3^2} \]

- The unit vector in the direction of \( \mathbf{v} \) is given by
  \[ \mathbf{u} = \frac{\mathbf{v}}{||\mathbf{v}||} \]

Let \( \mathbf{u} = \langle u_1, u_2, u_3 \rangle \) and \( \mathbf{v} = \langle v_1, v_2, v_3 \rangle \) and \( \mathbf{w} = \langle w_1, w_2, w_3 \rangle \)

- Dot Product: \( \mathbf{u} \cdot \mathbf{v} = u_1 v_1 + u_2 v_2 + u_3 v_3 = ||\mathbf{u}|| ||\mathbf{v}|| \cos(\theta) \), where \( \theta \) is the angle between \( \mathbf{u} \) and \( \mathbf{v} \).
  
  **Major Property:** The vectors \( \mathbf{u} \) and \( \mathbf{v} \) are orthogonal (perpendicular) if and only if \( \mathbf{u} \cdot \mathbf{v} = 0 \).

- Cross Product: \( \mathbf{u} \times \mathbf{v} = (u_2 v_3 - u_3 v_2) \mathbf{i} - (u_1 v_3 - u_3 v_1) \mathbf{j} + (u_1 v_2 - u_2 v_1) \mathbf{k} \)

  **Major property:** \( \mathbf{u} \times \mathbf{v} \) is orthogonal to both \( \mathbf{u} \) and \( \mathbf{v} \)
  
  **magnitude of cross product:** \( ||\mathbf{u} \times \mathbf{v}|| = ||\mathbf{u}|| ||\mathbf{v}|| \sin(\theta) \)

Summary of Lines, Planes, and Surfaces

- **Lines**
  Let \( P(x_1, y_1, z_1) \) be a point on the line and \( \mathbf{v} = \langle a, b, c \rangle \) be the direction vector.

  **parametric equations for the line:** \( x = x_1 + at, \ y = y_1 + bt, \ z = z_1 + ct. \)
  
  **symmetric equations for the line (provided \( a, b, c \neq 0 \):)** \( \frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c} \)

- **Planes**
  Let \( P(x_1, y_1, z_1) \) be a point on the plane and \( \mathbf{n} = \langle a, b, c \rangle \) be a normal vector.

  **equation in standard form:** \( a(x - x_1) + b(y - y_1) + c(z - z_1) = 0 \)
  
  **equation in general form:** \( ax + by + cz + d = 0 \)

- **Surfaces** Finding the equation for surfaces and sketching them generally requires finding the traces on various planes and fitting them together. Too many possibilities to list but but you can see the table on pages 765-766 in the text.