• $C^n$ is the vector space of all $n$-vectors with complex (or real) terms.

• Definition of Eigenvalue and Eigenvector

Let $A$ be an $n$ by $n$ matrix with real components. The number $\lambda$ (real or complex) is called an eigenvalue of $A$ if there is a nonzero vector $v$ in $C^n$ such that

$$Av = \lambda v$$  \hspace{1cm} (1)

The vector $v \neq 0$ is called an eigenvector of $A$ corresponding to the eigenvalue $\lambda$.

• Theorem 1 from 6.1

Let $A$ be an $n$ by $n$ matrix. Then $\lambda$ is an eigenvalue of $A$ if and only if

$$\det(A - \lambda I) = p(\lambda) = 0$$ \hspace{1cm} (2)

proof on the board

• Definition of Characteristic equation and polynomial

Equation (2) is called the characteristic equation of $A$ and $p(\lambda)$ is called the characteristic polynomial.

• Fundamental Theorem of Algebra.

Counting multiplicities, every $n$ by $n$ matrix has $n$ eigenvalues.

• Theorem 2 from 6.1

Let $\lambda$ be an eigenvalue for the $n$ by $n$ matrix $A$ and let

$$E_\lambda = \{ v : Av = \lambda v \}.$$ \hspace{1cm} (3)

Then $E_\lambda$ is a subspace of $C^n$ and is called the eigenspace of $A$ corresponding to $\lambda$.

• The geometric multiplicity of an eigenvalue is the dimension of the eigenspace corresponding to that eigenvalue.

• The algebraic multiplicity of an eigenvalue is the number of times it is a root of the characteristic equation.

• Process of finding Eigenvalues and Eigenvectors

1. Find $p(\lambda) = \det(A - \lambda I)$.
2. Find the roots $\lambda_1$, $\lambda_2$, $\ldots$, $\lambda_m$ of $P(\lambda) = 0$.
3. Corresponding to each eigenvalue $\lambda_i$, solve the homogenous system $(A - \lambda_i I)v = 0$.

• Examples: Find the eigenvalues and eigenspaces of the given matrix. If the algebraic multiplicity of an eigenvalue is greater than 1, calculate its geometric multiplicity.

$$\begin{pmatrix} 4 & 2 \\ 3 & 3 \end{pmatrix} \quad \begin{pmatrix} 3 & -5 \\ 1 & -1 \end{pmatrix} \quad \begin{pmatrix} 5 & 1 \\ 0 & 5 \end{pmatrix} \quad \begin{pmatrix} 2 & 2 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$