

1. Use the integral test to show that the harmonic series (below) diverges.

(10 pts)

$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$$

2. Show that the alternating harmonic series (below) converges. State the test you use and show all parts of the test.

(10 pts)

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$$

3. Classify the alternating harmonic series as absolutely convergent, conditionally convergent or divergent. (5 pts)

4. Find the general term for the n'th partial sum for the series below and determine whether the series converges or diverges. If it converges, then state it's sum. (10 pts)

$$\sum_{k=1}^{\infty} \left(\frac{1}{k} - \frac{1}{k+1} \right)$$

5. In this problem we investigate the sequence $\left\{ \frac{|x|^n}{n!} \right\}_{n=0}^{\infty} = \{a_n\}_{n=0}^{\infty}$ (15 pts)

(a) Use the ratio of successive terms to conclude that the sequence is eventually decreasing.

(b) Why must the sequence converge?

(c) What is the radius of convergence of the power series

$$1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^k}{k!} + \dots$$

6. Determine whether the series converges and if so, find its sum. Justify your answer. (20 pts)

(a)
$$\sum_{k=1}^{\infty} (-1)^{k-1} \frac{7}{6^{k-1}}$$

(b)
$$\sum_{k=1}^{\infty} \frac{1}{5 + 4^{-k}}$$

7. Classify the series as absolutely convergent, conditionally convergent, or divergent. You need not rigorously prove your conclusion but must provide sound reasoning.
$$\sum_{k=1}^{\infty} \frac{(-1)^k (k^2 + 1)}{k^3 + 3}$$
 (10 pts)

8. Verify that the Maclaurin series for $\ln(1 + x)$ is the power series (10 pts)

$$\sum_{k=1}^{\infty} (-1)^{k+1} \frac{x^k}{k} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$$

9. What is the radius of convergence and interval of convergence for the Maclaurin series in the previous problem. (10 pts)

10. **Bonus (5 points)** Why is it not surprising that the interval of convergence does not include $x = -1$.