1. Here we investigate the improper integral \( \int_{0}^{1} \ln x \, dx \). (16 pts)

(a) What makes the above definite integral improper?

(b) Use integration by parts to show that \( \int \ln x \, dx = x \ln(x) - x + C \).

(c) Use part (b) to express the improper integral \( \int_{0}^{1} \ln x \, dx \) as a limit.

(d) Does the improper integral converge or diverge. If it converges, to what does it converge. Hint: you will need to apply L'Hopital's rule to the expression \( \lim_{x \to 0^+} x \ln(x) = \lim_{x \to 0^+} \frac{\ln x}{1/x} \).

2. (a) Confirm graphically or algebraically that \( \frac{1}{x^2 + 3x + 2} \leq \frac{1}{x^2} \) for \( x \geq 1 \). (4 pts)

(b) Evaluate the integral the improper integral \( \int_{1}^{\infty} \frac{1}{x^2} \, dx \). (4 pts)

(c) Use parts (a) and (b) to conclude whether \( \int_{1}^{\infty} \frac{1}{x^2 + 3x + 2} \, dx \) converges or diverges. (4 pts)

3. Express the following statements as a differential equation. Be sure to state what each variable stands for and whether any constants are positive or negative. (8 pts)

(a) The rate at which the temperature of a warming turkey increases in an oven is proportional to the difference between the oven temperature and the temperature of the turkey.

(b) The amount of a drug that is present in the blood stream tends to decrease at a rate that is proportional to the amount present.
4. At time $t = 0$, a tank contains 25 ounces of salt dissolved in 50 gal of water. Then brine containing 4 ounces of salt per gallon of brine is allowed to enter the tank at a rate of 2 gal/min and the mixed solution is drained from the tank at the same rate. Let $y(t)$ denote the amount of salt in the tank at time $= t$.

(a) Set up the initial value problem describing the rate of change in $y$ with respect to time.

(b) Solve the initial value problem for $y(t)$.

(c) As $t \to \infty$ what does the concentration of salt in the tank tend towards.

5. A colony of the bacterium E. coli grows at a relative rate of 80 percent per hour when placed in a nutrient culture. Let $y = y(t)$ be the number of cells that are present after 100 cells are placed in the culture.

(a) Find an initial value problem whose solution is $y(t)$.

(b) Find a formula for $y(t)$, where all constants are assigned a numerical value.

(c) How long does it take for the number of cells to double.