DESIGNING A ROLLER COASTER

You have been hired by the Two Eagles Over Polson to help with the design of their new roller coaster near Flathead Lake.

Specifications

A sketch of the desired path design of the proposed coaster is given below (straight stretch, no turns).

A preliminary analysis shows that the height of the track given by the function $H(x)$ in meters by

$$H(x) = \frac{T}{2} \sin(x/10) + Ax + \frac{(x - 87.5)^2 T}{2(87.5^2)} - \frac{T}{2},$$

where $A$ and $T$ are constants, will satisfy the Two Eagles Over Polson requirements.

Your Obligations

Construct your own roller coaster design by taking $T$ to be the sum of the digits in your student ID number. Determine the constant $A$ by requiring the track to end at ground level, exactly 175 meters from the start. It is alright if your track goes below ground.

Figure out the following data for your report.

1. Where is the path increasing and decreasing? (Give your answer in terms of distance in m along the ground from the start.)

2. Locate the tops and the bottoms of the path.

3. Where is the path increasing at an increasing rate, increasing at a decreasing rate, decreasing at an increasing rate, and decreasing at a decreasing rate?

4. For each fall, where is the steepest descent and how steep is the angle at that point?

5. Draw the graph of the slope of the path versus distance along the ground from the start.

6. Draw the graph for the rate of change of the slope versus distance along the ground from the start.

7. The thrill of the coaster is defined as the sum of the angle of steepest descent in each fall in radians + number of tops from the start. Calculate the thrill of your path.

8. The amount of material needed for a support is the square of the height of the support. (So, for example, a support that is 20 meters high would require $20^2 = 400$ meters of material.) Find the amount of material needed for the support in your path, if there is to be a support at every 10 meters. If the path is on or below ground level at a support site, then no support is needed there.