Section 3.4 Optimization

General Strategy

1. Draw a picture if possible describing the problem.

2. Assign variables to the different quantities being described in the problem.

3. Write a primary equation describing the quantity to be maximized or minimized.

4. Reduce the primary equation to one having a single independent variable. This may involve the use of secondary equations relating the independent variables of the primary equation.

5. Determine a feasible domain of the primary equation. Often certain variables are only positive. The problem should make sense.

6. Differentiate the function describing the primary equation. Set this equal to zero or undefined and solve for the independent variable.

7. Determine which, if any, of the above solutions produces the desired result.

8. Ask yourself: Does this answer make sense?

- Example 1
  Find the largest possible rectangular region that can be enclosed using 120 m of fencing.
  
  1. Draw a picture
  2. Let \( x \) be the length of the rectangle and \( y \) be the width.
  3. We want to maximize area which we will denote by \( A \).

\[
A = xy \quad \text{this is the primary equation}
\]

Notice this equation involves two independent variables \( x \) and \( y \).

4. Use the secondary equation \( 2x + 2y = 120 \) to get one independent variable in terms of the other. Specifically let \( y = (120 - 2x)/2 = 60 - x \), and the primary equation becomes

\[
A = x(60 - x) \quad \text{the primary equation in one variable.}
\]

5. We want area to be positive so it is necessary that \( x \geq 0 \) and \( x \leq 60 \). This makes sense.

6. \( dA/dx = 60 - 2x \), setting this equal to zero we get \( x = 30 \).

7. It is easily shown by the first or second derivative test that \( A(30) = 900 \) is a relative maximum.

    Compare to \( A(0) \) and \( A(60) \) and it is an absolute maximum for \( 0 \leq x \leq 60 \).

8. The largest rectangular region that can be enclosed using 120 m of fencing is 900 m\(^2\) and results from the rectangle being a square with sides 30 m. This makes sense.

- Example 2
  Find the dimensions of the largest rectangle that can be inscribed in a semicircle of radius 4.

- Example 3
  An open rectangular box is to be made from a rectangular piece of metal 16 in by 10 in by cutting equal squares from each corner and folding up the sides. Determine the maximum volume of the box.