Math 141-002 Exam III (SPRING 2010)

This test is a closed-book test and no calculator is allowed. However, you are allowed a photocopy of page 644 in your textbook. Write your name below. Be sure to look at all problems before deciding which to do first. Note that some problems are easier than others. You may use the backs of the pages if you need additional space. You have 50 minutes to work.

Be sure to show your work.

Name: _____________________________

Instructor: Amos Ong M T W F 12:20 PM - 1:10 PM

1. [30 Points]
   (a) Sum the series \( \sum_{n=1}^{\infty} \frac{6}{n(n+2)} \).
   (b) Using geometric series, re-write \( 0.412 = 0.412121212 \ldots \) as a fraction.

2. [30 Points] Determine whether the following series converges conditionally, converges absolutely, or diverges.
   (a) \( \sum_{n=0}^{\infty} \frac{(-1)^n n}{2n+3} \)
   (b) \( \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}} \)
   (c) \( \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} \)

3. [20 Points] Find the third degree Maclaurin polynomial of \( f(x) = \tan(4x) \).

4. [40 Points] Determine the convergence/divergence of the following series:
   (a) \( \sum_{n=0}^{\infty} \frac{4^n}{3^n+4} \) (Comparison)
   (b) \( \sum_{n=0}^{\infty} \frac{(-3)^n}{n!} \)
   (c) \( \sum_{n=0}^{\infty} n e^{-n^2} \) (Integral)
   (d) \( \sum_{n=0}^{\infty} \left( \frac{n^2 + 1}{3n + 2n^2} \right)^n \) (Root Test)

5. [30 Points] Find the interval of convergence of each of the following power series:
   (a) \( \sum_{n=0}^{\infty} \frac{(x-2)^n}{n5^n} \)
   (b) \( \sum_{n=0}^{\infty} \frac{x^n}{n!} \)