Finals — Math 141 - Spring 2004

1. Write the following functions as power series centered at the given \( c \).
   (a) \( \frac{2}{2-x} \), \( c = 0 \)  
   (b) \( \frac{3}{x-3} \), \( c = 4 \)

2. Write the following equations in standard forms and determine which conic section they are:
   (a) \( 12x^2 + 20y^2 - 12x + 40y = 37 \)
   (b) \( 3x^2 - 2y^2 - 6x - 12y = 27 \)
   (c) \( y^2 - 6x = 9y \)

3. (a) Given that a hyperbola has vertices at \((0, \pm 5)\) and asymptotes \( y = \pm 3x \), find the equation in standard form. Where are the focii?

   (b) Given that a parabola has vertices at \((0, 4)\) and directrix at \( y = -2 \), find the equation in standard form. Where is the focus? What is the length of its latus rectum?

   (c) Given that an ellipse has vertices at \((\pm 10, 0)\) and has eccentricity \( \frac{3}{5} \), find the equation in standard form. Where are the foci?

4. Given the parametric equations
   \[ x = \frac{1}{2t + 1}, \quad y = \frac{1}{t^2 - 2t}. \]
   (a) Write down the corresponding Cartesian equation of the curve.

   (b) Find the slope of tangent at \( t = 1 \).

   (c) Find the arc-length of the curve from \( t = 1 \) to \( t = 3 \) (do not attempt the find the definite integral by hand, use your \texttt{fnInt} function on your calculator!)

5. (a) Approach the origin from different directions and find the limit of
   \[ \lim_{(x,y) \to (0,0)} \frac{2x^2 - y^2}{x^2 + y^2} \]
   Does it mean the function \( f(x, y) = \frac{2x^2 - y^2}{x^2 + y^2} \) is not continuous at some point? Explain briefly.
(b) Evaluate the following limit
\[
\lim_{(x,y)\to(4,3)} \frac{25xy}{6x^2 + 6y^2}
\]

6. Determine the convergence or divergence of the series.

(a) \[\sum_{n=1}^{\infty} \frac{1}{3^n - 4}\]

(b) \[\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^n\]

(c) \[\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n} - 1}\]

(d) \[\sum_{n=1}^{\infty} \frac{n^n}{n!}\]

7. Evaluate

(a) \[\lim_{x\to0} \frac{\arcsin x}{2x}\]

(b) \[\lim_{x\to1} \frac{\ln x}{x^2 - 1}\]

8. Evaluate the following indefinite integrals

(a) \[\int 3xe^{4x} \, dx\]

(b) \[\int x^4 \ln x \, dx\]

(c) \[\int \cos^6 x \sin^3 x \, dx\]

(d) \[\int \tan^3 x \sec x \, dx\]

(e) \[\int \frac{3}{x^2 + x - 2} \, dx\]

9. (a) Find the volume of the region bounded by \(y = x^2 - x^3\), \(y = 0\) between \(x = 0\), \(x = 1\) when generated about the \(y\)-axis.

(b) Find the area of the region bounded between \(y = \sin x\) and \(y = \cos x\) from \(x = \frac{\pi}{4}\) to \(x = \frac{\pi}{2}\).

**Bonus:** Find the volume of the intersection of two unit cylinder whose axes intersect perpendicularly.