Math 140 Calculus – Final Review

1. In which year did Galileo Galilei died? When was Issac Newton born? Who else discovered Calculus besides Newton?

2. Find limit analytically:

   (a) $\lim_{x\to 2} \frac{x^2-4}{x-2}$ (b) $\lim_{x\to 2^+} x^2+e^{x-2}$

   (c) $\lim_{x\to 2} \frac{\sqrt{6-x}-2}{2-x}$ (d) $\lim_{x\to 2^+} \frac{x-2}{\sqrt{x-2}}$

   (e) $\lim_{x\to 2^+} \frac{1-x}{2-x}$ (f) $\lim_{x\to +\infty} \frac{x^4-2x^2-10}{2-x-4x^4}$

   (g) $\lim_{x\to 0} \ln\left(\frac{\sin x}{x}\right)$ (h) $\lim_{x\to 0} \left(\frac{\cos x-1}{x}\right)^2$

3. Using the definition of derivatives, find $f'(x)$ for

   (a) $f(x)=\frac{1}{x}$ (b) $f(x)=\sqrt{x-2}$ (c) $f(x)=\frac{1}{x^2}$ (d) $f(x)=x^3$

4. Find equation of tangent line of $f(x)$ at $x=a$ for the following

   (a) $f(x)=\frac{1}{x}, \quad x=2$ (b) $f(x)=\sqrt{x-2}, \quad x=6$ (c) $f(x)=\sin x, \quad x=\frac{\pi}{3}$

5. Find $\frac{dy}{dx}$ for the following implicit functions

   (a) $x^2+xy^2+e^{xy}=2003$ (b) $x^2+y^3=3x^2y$

   (c) $x-3e^{xy}=0$ at $(3,0)$ (d) $\sqrt{x}+\sqrt{y}=4$ at $(9,1)$

6. The following functions satisfy the conditions for either the MVT (if not Rolle's theorem), find the value of c in the conclusion of MVT (Rolle's theorem):

   (a) $f(x)=\sin x,\ x\in[\pi,2\pi]$ (b) $f(x)=x^3-x,\ x\in[-1,1]$

   (c) $f(x)=5-x^2,\ x\in[-2,2]$ (d)$f(x)=x^2,\ x\in[-1,2]$

7. A spherical snowball is melting at the rate of 2π cubic inches per minute. At what rate is the radius decreasing when the radius is 3 inches? (Hints: $V=\frac{4}{3}\pi r^3$).

8. A 10-foot ladder is leaning against the wall. The bottom of the ladder is being pulled away horizontally at a speed of 3 feet per sec. Calculate how fast the top of the ladder is falling at the instant the top of the ladder is 6 feet above the ground.

9. The sum of two positive numbers is 15, what is the smallest sum of their fourth powers.

10. A box with surface area 120 square feet has its length equal twice its width. What dimension of this box has a maximum volume?

11. Two rectangular adjacent field share one side of their fence. If the total length of fence used is only 700 feet, what dimension of the field has a maximum area?

12. A factory makes cylindrical cans of volume $500cm^3$. If they want to use the least amount of metal, what should they make the diameter and height of the can? If the metal for the top and bottom of the can costs half as much as the metal for the sides, what should the dimensions of the can be to minimize the cost?

13. Determine on which intervals the following functions are increasing or decreasing. Concave up or concave down. Use the 2nd Derivative Test to test all its critical numbers.

(a) $y = x^3 - x^2$ (b) $f(x) = \dfrac{x-1}{x-2}$

(c) $y = xe^x$ (d) $y = 5x^3 - 3x^5$

14. Sketch the graph of the following by analysing its first and second derivatives, domain, intercepts, asymptotes, symmetry, extremas and concavity.

(a) $f(x) = \dfrac{x^2}{x^2+3}$ (b) $f(x) = \dfrac{5}{1+e^{-x}}$ (c) $f(x) = (x+1)(x+2)(x+4)$

15. Use the Fundamental Theorem of Calculus to find the derivatives of

(a) $F(x) = \int_0^x \sqrt{1+t^3}\,dt$ (b) $F(x) = \int_0^{\sqrt{x}} \sqrt{1+t^3}\,dt$ (c) $F(x) = \int_x^{\sqrt{x}} t\sin t\,dt$

(d) $F(x) = \int_x^1 e^{t^2}\,dt$ (e) $F(x) = \int_1^{5\sin x} s\sqrt{1+s}\,ds$ (f) $F(x) = \int_3^{x^2} s\sqrt{1+s}\;ds$

16. Using n equal subintervals, approximate the definite integrals of the following by a sum (in sigma notations) by evaluating the right-end point of each sub-interval:

(a) $\int_1^3 (x^2+x)\;dx$ (b) $\int_0^2 7x+1\;dx$ (c) $\int_0^1 1-x^3\;dx$

Find their limits as n goes to infinity.

17. Evaluate the following

(a) $\int_2^{2e} \left(x - \frac{1}{x}\right)\;dx$ (b) $\int x^4\sec^2(x^5)\;dx$ (c) $\int x^2\cosh(x^3)\;dx$

(d) $\int \frac{(\ln y)}{y}\;dy$ (e) $\int \frac{2x+1}{(x^2+x)^2+1}\;dx$ (f) $\int \frac{2x+1}{\sqrt{1-(x^2+x)^2}}\;dx$

(g) $\int \frac{1}{x^2+6x+10}\;dx$ (h) $\int \frac{2^{1/x^2}}{x^3}\;dx$ (i) $\int \frac{\cos x}{\sqrt{1+\sin^2 x}}\;dx$

(j) $\int \frac{e^x+2x}{x^2+e^x}\;dx$ (k) $\int_0^1 x^2 e^{x^3}\;dx$ (ℓ) $\int \frac{1}{x\ln x}\;dx$

18. Find the general solution of the following DE:

(a) $y' = \dfrac{x^2+2}{3y^2}$ (b) $(2+x)y' = 5y$ (c) $(1+x^2)y' = (1+y^2)$

19. Solve the following DE with the given initial conditions:

(a) $y' = -\dfrac{1}{\sqrt{xy}}$, $y(2) = 2$

(b) $y' = -5y$, $y(0) = 100000$

(c) $y' = -2(y-60)$, $y(1) = 70$