1. 1642, Leibniz, Siddhanta-Shiromani

2. (a) 4 (b) 5 (c) $\frac{1}{4}$ (d) 0
   (e) $+\infty$ (f) $-\frac{1}{4}$ (g) $\ln 1 = 0$ (h) 0

3. $f'(a) = \lim_{h\to 0} \dfrac{f(a+h)-f(a)}{h}$ if it exists.

   (a)

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{\frac{1}{x+h}-\frac{1}{x}}{h} \\ &= \lim_{h\to 0} \frac{x-(x+h)}{hx(x+h)} \\ &= \lim_{h\to 0} \frac{-1}{x(x+h)} = \frac{-1}{x^2} \end{aligned}$$

   (b)

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{\sqrt{x+h-2}-\sqrt{x-2}}{h} \\ &= \lim_{h\to 0} \frac{(x+h-2)-(x-2)}{h(\sqrt{x+h-2}+\sqrt{x-2})} \\ &= \lim_{h\to 0} \frac{1}{(\sqrt{x+h-2}+\sqrt{x-2})} \\ &= \frac{1}{2\sqrt{x-2}} \end{aligned}$$

   (c)

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h} \\ &= \lim_{h\to 0} \frac{x^2-(x+h)^2}{hx^2(x+h)^2} \\ &= \lim_{h\to 0} \frac{-2xh-h^2}{hx^2(x+h)^2} = -\frac{1}{x} \end{aligned}$$

4. (a) $y-\frac{1}{2} = -\frac{1}{4}(x-2)$ (b) $y-2 = \frac{1}{4}(x-6)$
   (c) $y-\frac{\sqrt{3}}{2} = \frac{1}{2}(x-\pi/3)$

5. (a) $\dfrac{dy}{dx} = \dfrac{-(ye^{xy}+y^2+2x)}{x(2y+e^{xy})}$
   (b) $\dfrac{dy}{dx} = \dfrac{2x(3y-1)}{3(y^2-x^2)}$
   (c) $y' = \frac{1}{9}$
   (d) $y' = -\frac{1}{\sqrt{x}}\sqrt{y} = -\frac{1}{3}$

6. (a) $c = \frac{3\pi}{2}$ (b) $c = \pm\frac{1}{\sqrt{3}}$ (c) $c = 0$
   (d) $c = \frac{1}{2}$ (Rolle)

7. $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$. $\frac{dV}{dt} = 2\pi, r = 3 \Rightarrow \frac{dr}{dt} = \frac{1}{18}$

8. $x^2+y^2 = 10^2$,
   $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0 \Rightarrow \frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt} = -4$

9. $x+y = 16$ (Secondary equation)
   $S = x^4+y^4$ (Primary Equation)
   $S = x^4+(16-x)^4, 0 < x < 16$
   $S' = 4x^3-4(16-x)^3 = 0 \Rightarrow$ x=y=8
   $S''(8) > 0$

10. $h = \dfrac{120-4w^2}{6w}, V = \dfrac{w}{3}(120-4w^2)$
    V is a maximum when $w = \sqrt{10}$.

11. $700 = 3y+4x$
    $A = 2xy = 2x(700-4x)/3$
    A is a max. when $x = \frac{700}{8}$

12. $\pi r^2 h = 500, S = 2\pi r^2 + 2\pi rh$
    $C = \pi r^2 k + 2\pi r^2 h$
    S is a minimum when $r = \sqrt[3]{\frac{250}{\pi}}$.
    C is a minimum when $r = \sqrt[3]{\frac{500}{\pi}}$.

13.

	Inc	Dec
a)	$(\infty,0),(\frac{2}{3},\infty)$	$(0,\frac{2}{3})$
b)	never	$(-\infty,2),(2,\infty)$
c)	$(-1,\infty)$	$(-\infty,-1)$
d)	$(-\infty,-1),(0,1)$	$(-1,0),(1,\infty)$

	CU	CD
a)	$(\frac{1}{3},\infty)$	$(-\infty,\frac{1}{3})$
b)	$(-\infty,2)$	$(2,\infty)$
c)	$(-2,\infty)$	$(-\infty,-2)$
d)	$(-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})$	$\left(-\infty,-\frac{1}{\sqrt{2}}\right),\left(\frac{1}{\sqrt{2}},\infty\right)$

14. (a) $f'(x) = -\frac{6x}{(x^2+3)^2}, f''(x) = \frac{-18(3-x^2)}{(x^2+3)^3}$.
    (b) $f'(x) = -\frac{5e^{-x}}{(1+e^{-x})^2}$,
    $f''(x) = \frac{5e^{-x}(e^{-x}-1)}{(1+e^{-x})^3}$.
    (c) $f'(x) = 3x^2+14x+14, f''(x) = 6x+14$

I could only do so much for now Hope I could do more later ...