Math 141 Calculus – Review

1. Write the following functions as power series centered at the given c.
   (a) \( \frac{3}{2 + x}, \quad c = 0 \)  
   (b) \( \frac{x}{x - 3}, \quad c = 2 \)  
   (c) \( \frac{11}{x - 2}, \quad c = 0 \)  
   (d) \( \frac{1}{x - 2}, \quad c = 3 \)  
   (e) \( \frac{2}{1 + x}, \quad c = 0 \)  
   (f) \( \frac{1}{x}, \quad c = -2 \)

2. Write the following equations in standard forms and determine if they are parabola, hyperbola or ellipse:
   (a) \( 12x^2 + 20y^2 - 12x + 40y - 37 = 0 \)  
   (b) \( 36x^2 + 9y^2 + 48x - 36y + 43 = 0 \)  
   (c) \( 9x^2 - 4y^2 + 54x + 8y + 78 = 0 \)  
   (d) \( 3x^2 - 2y^2 - 6x - 12y - 27 = 0 \)  
   (e) \( y^2 - 4x = 4y \)

3. Find equation of the following conic sections:
   (a) parabola; vertex=(0,4), directrix: \( y = -2 \)  
   (b) hyperbola; asymptotes \( y = \pm \frac{3}{5}x \), vertices=(\( \pm 5, 0 \))  
   (c) ellipse; \( e = \frac{3}{5}, \) vertices=(\( 3 \pm \frac{5}{2}, 2 \))  
   (d) hyperbola; vertices=(2 \( \pm \frac{5}{2}, -3 \)), focii=(\( 2 \pm 4, -3 \))

4. Find the area between the two curves:
   (a) \( f(x) = x, \ g(x) = x^3 \)  
   (b) \( f(x) = \tan x, \ g(x) = \sqrt{x}, \ x \in [0, \frac{\pi}{4}] \)  
   (c) \( f(x) = x^2, g(x) = -x^4 \)  
   (d) \( f(x) = \cos x, g(x) = 2x^2, \ x \in [-1, 1] \)

5. Find the volume generated by the region bounded by the equations about the given axis:
   (a) \( y = x, y = x^2 \) about the y-axis  
   (b) \( y = \tan x, y = 1 \) about the x-axis  
   (c) \( y = x^2, y = 8 - x^2 \) about the line \( y = 4 \)  
   (d) \( y = x^2, y = 8 - x^2 \) about the y-axis

6. Find the slope and concavity of
   (a) \( x = 2 \cos t, \quad y = 3 \sin t \)  
   (b) \( x = 2t^2, y = 3t^2 + 7t, \ \text{at} \ t = 1 \)  
   (c) \( x = 1 - t, \quad y = t^2 \)  
   (d) \( y = 2 \sin 2t, \quad x = 3 \sin t \)  
   (e) arclength of (b) from \( t = -1 \) to \( t = 1 \)  
   (f) arclength of (c) from \( t = 0 \) to \( t = 1 \)

7. Determine the convergence or divergence of the series:
   (a) \( \sum_{n=1}^{\infty} \frac{3^n}{5^n + 4} \)  
   (b) \( \sum_{n=1}^{\infty} \left( \frac{1}{n + 1} \right)^n \)  
   (c) \( \sum_{n=3}^{\infty} \frac{(-1)^{n+1}}{\ln(n - 1)} \)  
   (d) \( \sum_{n=1}^{\infty} \frac{1}{3^n - 4} \)  
   (e) \( \sum_{n=1}^{\infty} \frac{n + 1}{n} \)  
   (f) \( \sum_{n=1}^{\infty} \frac{n!}{n^n} \)

8. Determine the radius of convergence of
   (a) \( \sum_{n=1}^{\infty} (-3)^n x^n \)  
   (b) \( \sum_{n=1}^{\infty} \frac{(2x)^n}{n^2} \)  
   (c) \( \sum_{n=3}^{\infty} \frac{(3x)^n}{n!} \)  
   (d) \( \sum_{n=1}^{\infty} \frac{n!x^n}{(2n)!} \)  
   (e) \( \sum_{n=1}^{\infty} \frac{(x + 3)^n}{10^n} \)  
   (f) \( \sum_{n=1}^{\infty} \frac{(x + 2)^n}{n!} \)
9. Conditionally conv.? Absolutely conv.? Or Div?
   (a) \( \sum_{n=2}^{\infty} \frac{(-1)^{n+3}n^2}{(n-1)^2} \)  
   (b) \( \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}} \)  
   (c) \( \sum_{n=3}^{\infty} \frac{(-1)^n}{\ln(n+1)} \)

10. Evaluate the following limits analytically
   (a) \( \lim_{x \to 0} \frac{\arcsin x}{2x} \)  
   (b) \( \lim_{x \to -1} \frac{\ln x}{x^2 - 1} \)  
   (c) \( \lim_{x \to 0} \frac{\sin 5x}{3x} \)  
   (d) \( \lim_{x \to \infty} \frac{2e^x + e^{-x}}{3x^2 - y^2} \)  
   (e) \( \lim_{x \to 1} \frac{3x^2 - y^2}{x^2 + y^2} \)  
   (f) \( \lim_{x \to 1} \frac{3x^3 - 2y^3}{x^2 + y^2} \)

11. Evaluate the following indefinite integrals
   (a) \( \int 3x^2e^{-2x} \, dx \)  
   (b) \( \int \cos^5 x \sin^4 x \, dx \)  
   (c) \( \int 3x^2 \ln(x) \, dx \)  
   (d) \( \int \tan^2 x \sec^3 x \, dx \)  
   (e) \( \int \frac{1}{x^2 + 2x - 3} \, dx \)  
   (f) \( \int \frac{1}{x^2(x+1)} \, dx \)  
   (g) \( \int \cos 5x \cos 4x \, dx \)  
   (h) \( \int \sin 5x \sin 3x \, dx \)  
   (i) \( \int (4x + 2)e^{x^2+x} \, dx \)

12. Given the following polar equations, find the area of:
   (a) one petal of \( r = 6 \sin 2\theta \)  
   (b) one petal of \( r = 2 \cos 3\theta \)

13. Evaluate each of the improper integrals:
   (a) \( \int_{1}^{\infty} \frac{1}{x^2} \, dx \)  
   (b) \( \int_{0}^{\infty} \frac{1}{e^x} \, dx \)  
   (c) \( \int_{0}^{\infty} \frac{1}{1 + x^2} \, dx \)

14. Find \( f_x, f_y, f_{xx}, \) and \( f_{xy} \), of
   (a) \( f(x) = xe^y + \sin(xy) \)  
   (b) \( f(x) = \frac{x^2 + y^2 - 9}{y} \)  
   (c) \( f(x) = \arctan(x + y) \)

Reminder: You are allowed to bring in one A4 size cheat sheet handwritten on ONE side.

This review questions are typeset in a matter of hours with very little checking and it is very likely that there could be typos. If you see any major errors that has not been reported on Erratum on the course webpage, please email me (bwo1@psu.edu).