Topological Theory: The Three C’s
Continuity, Connectedness, Compactness

Continuity

**Def:** We say a function \( f : (X, T_1) \rightarrow (Y, T_2) \) is **continuous** if . . .

**Ex 1:** Let \( Y = \{0, 1\} \). Suppose the topology on \( Y \) is the **discrete topology**, \( \mathcal{P}(Y) \). Define the function \( f : \mathbb{R} \rightarrow Y \) by

\[
f(x) = \begin{cases} 
0 & \text{if } x < 0 \\
1 & \text{if } x \geq 0
\end{cases}
\]

Show that \( f \) may be continuous or discontinuous, depending upon the topology on \( \mathbb{R} \).
**Def:** Let \((X, T_1)\) and \((Y, T_2)\) be topological spaces. We say that a function 
\(f : (X, T_1) \to (Y, T_2)\) is a **homeomorphism** if . . .

1.

2.

3.

**Def:** We say two topological spaces \((X, T_1)\) and \((Y, T_2)\) are **homeomorphic** (written \(X \sim Y\)) if . . .

**Def:** A property \(P\) is called a **topological invariant** if . . .

**In-Class Exercise:**

Let \(T_1\) be the **discrete topology** on \(X\), i.e. \(T_1 = \mathcal{P}(X)\), and let \((Y, T_2)\) be any topological space. Show that \(f : (X, T_1) \to (Y, T_2)\) is continuous.