SEPARATION AXIOMS

Purpose of the 5 Axioms:

Def: Let \((X, T)\) be a topological space. The following are called separation axioms.

- We say \((X, T)\) is \(T_0\) if . . .

- We say \((X, T)\) is \(T_1\) if . . .
  1.
  2.

- We say \((X, T)\) is \(T_2\) (aka Hausdorff) if . . .
  1.
  2.
  3.

- We say \((X, T)\) is regular if . . .
  1.
  2.

- We say \((X, T)\) is normal if . . .
  1.
  2.
**Thm 1:** Normal $\implies$ regular $\implies T_2$ (Hausdorff) $\implies T_1$ $\implies T_0$

*Proof.* HW Assignment #11 Problem #1. (This is Exercise 3.23 in the book.)  

**Ex 1:** Determine which of the separation axioms are satisfied by each of the topological spaces $(X, \mathcal{T})$ below.

(a) $X = \{a, b\}$, $\mathcal{T} = \{X, \emptyset\}$

(b) $X = \{a, b\}$, $\mathcal{T} = \{X, \{a\}, \emptyset\}$

(c) $X = \mathbb{R}$, $\mathcal{T} =$ finite complement topology.

**Ex 2:** Let $(X, \mathcal{T})$ be the topological space in Ex 1c. Let $\{a_n\}$ be the sequence in $X$ given by $a_n = n$. Show that every real number is a limit point of this sequence.
Thm 2: If $X$ be a Hausdorff space and $Y$ is a compact subspace of $X$, then . . .

Proof. HW Assignment #11 Problem #2. (This is Exercise 3.26 in the book.) □

Thm 3: If $f : X \to Z$ is 1-1, onto, and continuous with $X$ compact and $Z$ Hausdorff, then . . .

Proof. HW Assignment #11 Problem #3. (This is Exercise 3.27 in the book.) □

Thm 4: Each of the separation axioms is . . .

HW Assignment #11 Problems:

1. Do 3.23 in the book.
3. Do 3.27 in the book.