RELATIVE NEIGHBORHOODS

Ex 1: Is the interval $(0, 1)$ open?

- 
- 

Goal for today:

Def: Let $A \subseteq \mathbb{R}^n$ and let $x \in A$. A neighborhood of $x$ relative to $A$ is . . .

Examples of relative neighborhoods:

1. Suppose $A$ is the cylinder depicted. Sketch neighborhoods of $x$ and $y$ relative to $A$.

\begin{center}
\includegraphics[width=0.3\textwidth]{cylinder.png}
\end{center}

2. Suppose $A = \{(x, 0) \in \mathbb{R}^2 : 1 \leq x < 3\}$. Sketch neighborhoods of $(2, 0)$ and $(1, 0)$ relative to $A$.

3. Let $A = \mathbb{R}^1 \subseteq \mathbb{R}^2$. Sketch a neighborhood of some $x \in A$ relative to $A$.

- Observe:
- In general,
Generalizations of the Crucial Definitions: (Memorize ASAP)

Let \( B \subseteq A \).

- A point \( x \in A \) is an **interior point** of \( B \) relative to \( A \) if . . .

- A point \( x \in A \) is an **exterior point** of \( B \) relative to \( A \) if . . .

- A point \( x \in A \) is a **limit point** of \( B \) relative to \( A \) if . . .

**Def:** Let \( B \subseteq A \). The set \( B \) is **open relative to** \( A \) if . . .

**Note:**

**Thm 1:** Suppose \( B \subseteq A \subseteq \mathbb{R}^n \). The set \( B \) is open relative to \( A \) if and only if . . .

*(Lots more room on next page.)*
**Thm 2:** Suppose $B \subseteq A \subseteq \mathbb{R}^n$. The set $B$ is closed relative to $A$ if and only if …
In-Class Exercise:
Ex 2.21. Give an example of sets $B \subseteq A \subset \mathbb{R}^3$ where $B$ is open relative to $A$ but not open in $\mathbb{R}^3$.

HW #4 Problems:
HW 2.19. Show that any set $A$ is both open and closed relative to itself.

HW 2.22. Show that $A$ is open relative to $X$ if and only if $X - A$ is closed relative to $X$. 