PRODUCT SPACES

Recall the Def: Let $A$ and $B$ be wo sets. The (Cartesian) product $A \times B$ is ...

Prop 1: Let $(X, \mathcal{T}_X)$ and $(Y, \mathcal{T}_Y)$ be topological spaces. Show that the set $\beta = \{U \times V : U \in \mathcal{T}_X$ and $V \in \mathcal{T}_Y\}$ is a basis for a topology on $X \times Y$.

Def: Let $(X, \mathcal{T}_X)$ and $(Y, \mathcal{T}_Y)$ be topological spaces. The product topology on $X \times Y$ is ...

True or False: Every open set $O$ in the product topology on $X \times Y$ can be written as $O = U \times V$, where $U \in \mathcal{T}_X$ and $V \in \mathcal{T}_Y$.

Examples:

(a) $\mathbb{R}_{\text{usual}} \times \mathbb{R}_{\text{usual}}$ is homeomorphic to ...

(b) Recall: $S^n$ is the n-dimensional unit sphere. 
    $S^1 \times \mathbb{N}$ is ...

(c) $S^1 \times [0,1]$ is ...

(d) $S^1 \times S^1$ is ...
Def: The projection map onto $X$ is . . .

Prop 2: The projection maps are . . .

Thm 1: If $(X, \mathcal{T}_X)$ and $(Y, \mathcal{T}_Y)$ are connected, then . . .

Lemma 1: Let $A_\alpha$ be a collection of connected topological spaces which have a point $x$ in common. Then . . .
**Thm 2:** If \((X, T_X)\) and \((Y, T_Y)\) are compact, then . . .

**Lemma 2:** If there is a basis \(\beta\) for \((X, T)\) such that every open cover of \(X\) which consists of elements of \(\beta\) has a finite subcover, then . . .
Note: If \((X, T_X)\) and \((Y, T_Y)\) have the fixed point property, then . . .


**HW Assignment #12 Problem:**

1. Do 3.29 in the book.