HOMEOMORPHISM

Def: The identity function $1_X$ on a set $X$ is . . .

Def: A function $f : X \rightarrow Y$ is called invertible if . . .

Recall some Thms from Math 311W:

• The set-theoretic inverse $f^{-1}$ of a function $f : X \rightarrow Y$ is itself a function iff . . .

• If $f^{-1}$ is a function, then it is a function \textit{from} $Y$ \textit{to} $X$ iff . . .

• Thus, a function $f : X \rightarrow Y$ is invertible iff . . .

Another Crucial Def:
We say a function $f : X \rightarrow Y$ is a homeomorphism if . . .

The spaces $X$ and $Y$ are then called . . .
**Ex 1:** Prove that the interval $(-1,1)$ is homeomorphic to $\mathbb{R}$.

**Ex 2:** Prove that $\{(x,y) \in \mathbb{R}^2 : x^2 + y^2 = 1\} - \{(1,0)\}$ is homeomorphic to the interval $(0,2\pi)$.

**Ex 3:** Explain why the function $f$ that cuts the strip below in half is not a homeomorphism.
Ex 4: Let $C = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}$. Explain why the function $f : [0, 2\pi) \to C$ which is given by $f(\theta) = (\cos \theta, \sin \theta)$ is not a homeomorphism.

Note that $f$ glues the ends of the interval to make a circle.

Thm: Topological equivalence is . . .
Def: Property $P$ is called a topological property (aka topological invariant) if ...

Ex 5: One example of a topological invariant is ...

Ex 6: Use Example 1 to find a geometric invariant (ie: a property that doesn’t change under rigid motions) that is not a topological invariant.

HW #5: Justify that $\mathbb{N} \subseteq \mathbb{R}$ is not homeomorphic to $\mathbb{Q} \subseteq \mathbb{R}$, even though they have the same cardinality.

   Hint: Suppose BWOC that there is a homeomorphism $f : \mathbb{Q} \to \mathbb{N}$.  