Math 429 Exam 1 Review

Definitions: Be prepared to define the following terms.

- neighborhood of $x \in \mathbb{R}^n$
- neighborhood of $x$ relative to a set $X \subseteq \mathbb{R}^n$
- interior point of a set $A$ in $\mathbb{R}^n$
- interior point of a set $A$ relative to a set $X \subseteq \mathbb{R}^n$.
- exterior point of a set $A$ in $\mathbb{R}^n$
- exterior point of a set $A$ relative to a set $X \subseteq \mathbb{R}^n$.
- limit point of a set $A$ in $\mathbb{R}^n$
- limit point of a set $A$ relative to a set $X \subseteq \mathbb{R}^n$.
- isolated point of a set $A$ in $\mathbb{R}^n$
- frontier point of a set $A$ in $\mathbb{R}^n$
- open set $A$ in $\mathbb{R}^n$
- open set $A$ relative to a set $X \subseteq \mathbb{R}^n$.
- closed set $A$ in $\mathbb{R}^n$
- closed set $A$ relative to a set $X \subseteq \mathbb{R}^n$.
- $Int(A)$, $Fr(A)$, $Ext(A)$, and $Cl(A)$
- bounded set $A$ in $\mathbb{R}^n$

Be prepared to use the following theorems:

- $Cl(A) = \{\text{limit points of } A\}$.
- $D^n(x, r)$ is open in $\mathbb{R}^n$.

- For any set $A$ in $\mathbb{R}^n$, the set $Cl(A)$ is closed in $\mathbb{R}^n$.
- A set $A$ is open in $\mathbb{R}^n$ iff $\mathbb{R}^n - A$ is closed in $\mathbb{R}^n$. (You should be able to prove this too!)
- A set $A$ is open relative to $X \subseteq \mathbb{R}^n$ iff $X - A$ is closed relative to $X$. (You should be able to prove this too!)
- If $A$ and $B$ are open (in $\mathbb{R}^n$ or relative to $X \subseteq \mathbb{R}^n$), then $A \cup B$ and $A \cap B$ are open (in $\mathbb{R}^n$ or relative to $X \subseteq \mathbb{R}^n$). (You should be able to prove this too!)
- If $A$ and $B$ are closed (in $\mathbb{R}^n$ or relative to $X \subseteq \mathbb{R}^n$), then $A \cup B$ and $A \cap B$ are closed (in $\mathbb{R}^n$ or relative to $X \subseteq \mathbb{R}^n$). (You should be able to prove this too!)
Problems: Be prepared to

- solve any problem like that in the homework or in-class exercises.
- classify up to topological equivalence a list of figures.
- find interiors, frontiers, and closures of some sets.
- provide an example of a set $B$ such that $B \subseteq A \subseteq \mathbb{R}^n$ for some given set $A$ and where $B$ is open relative to $A$ but not open in $\mathbb{R}^n$. 