NONHOMOGENEOUS LINEARODES WITH CONSTANT COEFFICIENTS
and the
SUPERPOSITION PRINCIPLE

Thm (The Superposition Principle): Let $y_1$ be a solution to $ay'' + by' + cy = f_1(t)$ and let $y_2$ be a solution to $ay'' + by' + cy = f_2(t)$. Then . . .

Ex 1: Find a particular solution to the ODE $y'' - 2y' - 3y = f_1(t) + f_2(t)$, where
(a) $f_1(t) = 5t + 2$ and $f_2(t) = e^{3t}$.

(b) $f_1(t) = 45t + 18$ and $f_2(t) = 7e^{3t}$.

Cor: A general solution for $ay'' + by' + cy = g(t)$ . . .

Ex 2: Find a general solution to the ODE $y'' - 2y' - 3y = 5t + 2$. 
Ex 3: Give the form of a particular solution to the ODE \( y'' - 2y' - 3y = g(t) \), where

(a) \( g(t) = 2t^2 e^{3t} + 7t \cos(5t) + e^t \sin t \).

(b) \( g(t) = 6e^{-t} \sin(2t) + 3t \cos(2t) + 9te^{-t} \).

Note: The Method of Undetermined Coefficients only works for

1.  
2.  

(a)  
(b)  
(c)  
(d)

Undetermined Coefficients won’t work when

1.  
2.
**Ex 4:** Find a particular solution to \( y'' + 4y = \csc t. \)

**Ex 5:** Suppose the third order ODE \( ay''' + by'' + cy' + dy = f(t) \) has corresponding characteristic equation
\[ (r - 4)(r + 3)^2 = 0. \]
Find the form of a particular solution if

(a) \( f(t) = e^{-2t}. \)

(b) \( f(t) = e^{4t}. \)

(c) \( f(t) = e^{-3t}. \)

(d) \( f(t) = \cos 2t \)

(e) \( f(t) = e^{-3t} + e^{4t} + \cos 2t \)

**Ex 6:** Find the general solution to the ODE \( y'' + 9y = \cos(3t). \)

*More room on next page!*