Recall: Suppose the function $g(t)$ is defined on $[0, \infty)$. The Laplace transform of $g$ is given by . . .

Ex 1: Compute $\mathcal{L}\{te^{-t}\}(s)$.

Ex 2: Use the table of Laplace transforms to compute $\mathcal{L}\{t^3e^{-t}\}(s)$.

Thm 1: The Laplace transform is a linear operator. That is,

1. $\mathcal{L}\{c g\} = \ldots$

2. $\mathcal{L}\{g_1 + g_2\} = \ldots$
Ex 3: Compute $L\{-7e^{\sqrt{2}t} + 4t^3e^{-t} - 13\cos(32t)\}(s)$.

Thm 2: If $L\{g\}(s)$ exists for $s > \alpha$, then . . .

Proof:

Ex 4: Find $L\{e^{2t}F(t)\}(s)$, where $F(t) = \begin{cases} 18 & \text{for } 0 \leq t < 10 \\ 24 & \text{for } 10 \leq t < \infty \end{cases}$.

Thm 3: Suppose $f(t)$ is continuous on $[0, \infty)$, $f'(t)$ is piecewise continuous on $[0, \infty)$, and both $f$ and $f'$ have exponential order $\alpha$. Then for $s > \alpha$, . . .
Thm 3 for higher order derivatives:

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Ex 5: Take the Laplace transform of both sides of the equation $3y'' + 5y' + 10y = t \cos t$.

Thm 4: Suppose $L\{f\}(s)$ exists for $s > \alpha$, then . . .

Ex 6: Compute $L\{t \cos(t)\}(s)$.
Laplace Transform Formulas

\[ \mathcal{L}\{1\}(s) = \frac{1}{s}, \quad s > 0 \]

\[ \mathcal{L}\{t^n\}(s) = \frac{n!}{s^{n+1}}, \quad s > 0 \]

\[ \mathcal{L}\{e^{at}\}(s) = \frac{1}{s-a}, \quad s > a \]

\[ \mathcal{L}\{\cos bt\}(s) = \frac{s}{s^2 + b^2}, \quad s > 0 \]

\[ \mathcal{L}\{\sin bt\}(s) = \frac{b}{s^2 + b^2}, \quad s > 0 \]

\[ \mathcal{L}\{e^{at}\cos bt\}(s) = \frac{s-a}{(s-a)^2 + b^2}, \quad s > a \]

\[ \mathcal{L}\{e^{at}\sin bt\}(s) = \frac{b}{(s-a)^2 + b^2}, \quad s > a \]

\[ \mathcal{L}\{e^{at}t^n\}(s) = \frac{n!}{(s-a)^{n+1}}, \quad s > a \]