EXACT EQUATIONS

Recap of First Order ODEs so far:

1. Separable:

2. Linear:

3. Exact:

Def: The equation $M(x, y)dx + N(x, y)dy = 0$ is an exact differential equation if there is a function $f$ such that

1.

2.

3.

Thm: The general solution of the exact equation above is . . .

Test for Exactness Thm:

Let $M$ and $N$ have continuous partial derivatives in a rectangle $R$. Then the differential equation $M(x, y)dx + N(x, y)dy = 0$ is exact in $R$ if and only if . . .

Ex 1: Are the following equations exact?

(a) $(xy^2 + x) dx + yx^2 dy = 0$
(b) \((xy + 1) \, dx + yx \, dy = 0\)

Solving Exact Equations

1. First, check to see if . . .

2. If the equation is not \underline{\hspace{1cm}}, then check to see if . . .

3. Use the fact that there exists a function \(f\) such that

\[ f_x(x, y) = M(x, y) \quad \text{and} \quad f_y(x, y) = N(x, y) \]

to find \(f\).

(a) **Method 1**: Exact implies . . .

(b) **Method 2**: Exact implies . . .

(c) **Alternate Method 2**: Exact implies . . .

**Ex 2**: Solve the ODE \((2xy - 3x^2) \, dx + (x^2 - 2y) \, dy = 0\).
**Ex 3:** Solve the IVP \((\cos x - x \sin x + y^2)dx + 2xydy = 0,\quad y(\pi) = 1.\)

*Hints:* (1) Use Alternate Method 2. (2) If you can’t guess the integral, you will need to use integration by parts.