Section 15.5

(Yes, we’re skipping 15.4 for the moment. We’ll come back to it.)

**PARAMETRIC SURFACES**

**Recall:** If the vector field $\mathbf{F}$ is not conservative, then to compute $\int_C \mathbf{F} \cdot d\mathbf{r}$, we must first ...

**Def:** A parametric surface is ...

Parametric Surfaces are like Parametric Curves:

**Ex 1:** Find parametric equations for the following surfaces.

(a) $z = x^2 + y^2$

(b) $3x + y - z = 7$
(c) The part of the surface \( z = x^2 + y^2 \) that is inside the cylinder \( x^2 + y^2 = 4 \).

(d) The part of \( 3x + y - z = 7 \) that is inside \( x^2 + y^2 = 9 \).

**Surfaces of Revolution:** A parameterization of the surface of revolution formed by revolving the graph of

- \( f(x) \) about the \( x \)-axis is . . .

- \( f(y) \) about the \( y \)-axis is . . .

- \( f(z) \) about the \( z \)-axis is . . .

**Ex 2:** Find a parameterization of the surface formed by revolving the graph of \( y = \sin x \), \( 0 \leq x \leq \pi \) about the \( x \)-axis.
Tangent and Normal Vectors to Parametric Surfaces:

A General Principle: Suppose a parametric surface $S$ is given by $\mathbf{r}(u, v) = (x(u, v), y(u, v), z(u, v))$. 

(a) Find two easy vectors that are tangent to $S$ at the point $\mathbf{r}(u_0, v_0)$.

(b) Find a normal vector to the surface $S$ at the point $\mathbf{r}(u_0, v_0)$.

Ex 3: Find the equation of the tangent plane to the paraboloid given by $\mathbf{r}(x, y) = (x, y, x^2 + y^2)$ at the point $(1, -2, 5)$. 
Surface Areas of Parametric Surfaces:

Recall:

Arc Length Formulas: 

Surface Area Formulas:

Ex 4: Use the latest formula to find the area of the portion of the paraboloid $z = x^2 + y^2$ inside $x^2 + y^2 = 4$. Hint: Use the parameterization from Example 1 c.