TANGENT PLANES AND NORMAL LINES

Def: Let $S$ be a smooth surface and let $P$ be a point on $S$. The **tangent plane** to $S$ at $P$ is ...

Def: Let $S$ be a smooth surface and let $P$ be a point on $S$. The **normal line** to $S$ at $P$ is ...

3 Ways to Describe a Surface:

1.

2.

3.
Recall: Suppose \((x_0, y_0)\) is a point on the level curve \(f(x, y) = c\). A 2-dimensional vector that is normal to the level curve \(f(x, y) = c\) at the point \((x_0, y_0)\) is . . .

Proof:
**Thm:** Let $F$ be differentiable at a point $P = (x_0, y_0, z_0)$ on a level surface $S$ of $F$. If $\nabla F(x_0, y_0, z_0) \neq (0, 0, 0)$, then

**Proof:**
Ex 1: Let \( f(x, y) = x^3 - 3x^2y + y^2 \).

(a) Find the tangent plane to the graph of \( f \) at the point \((1, -1, 5)\).

(b) Find a set of parametric equations for the normal line to the graph of \( f \) at \((1, -1, 5)\).

Ex 2: Find the equation of the tangent plane to the surface given by \( z = x^2 + y^2 \) at the point \( P = (1, 2, 5) \).
Ex 3: Consider the surfaces given by $z = x^2 + y^2$ and $x + y + 6z = 33$. Observe that the point $P = (1, 2, 5)$ is on the curve of intersection of these surfaces.

(a) Find a set of parametric equations for the tangent line to the curve of intersection at the point $P$.

(b) Find the angle of intersection between the surfaces at the point $P$. 
Ex 4: Consider the surfaces given by $x^2 + 2y^2 + 2z^2 = 20$ and $x^2 + y^2 + z = 4$.

(a) Find a set of parametric equations for the tangent line of the curve of intersection at the point $P = (0, 1, 3)$.

(b) Find the angle of intersection between the surfaces at the point $P$. 