DIFFERENTIALS

Ex 1: Suppose $\Delta x$ represents the change in $x$ (aka, “the increment of $x$”) and $\Delta y$ represents the change in $y$ (aka, “the increment of $y$”). Write a formula for the change $\Delta z$ in $z = f(x, y)$ (aka, “the increment of $z$”).

Def: Let $z = f(x, y)$. The total differential of $z$ is given by . . .

Informal Def: We say the function $z = f(x, y)$ is differentiable at $(a, b)$ if . . .

Thm 1: A function $z = f(x, y)$ is differentiable at $(a, b)$ if . . .

Thm 2: If a function $z = f(x, y)$ is differentiable at $(a, b)$, then . . .

Thm 2 again (this time, the contrapositive version):
Ex 2: Show that \( f(x, y) = \begin{cases} \frac{2x^2y^2}{\sqrt{x^4+y^4}} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases} \) is not differentiable at \((0, 0)\).
**Application: Error Analysis**

Use the following facts:

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**Ex 3:** Electrical power $P$ is given by $P = \frac{E^2}{R}$, where $E$ is the voltage and $R$ is the resistance. The possible percentage errors in measuring $E$ and $R$ are 2% and 3%, respectively. Approximate the maximum percentage error in calculating the power. Then approximate the maximum propagated error in calculating power if 200 volts is applied to a 4000-ohm resistor.
Ex 4: The radii of a frustum of a right circular cone are measured as $a = 5$ meters and $b = 15$ meters. In the process of measuring, 0.1 meter errors in $a$ and $b$ are made. Approximate the maximum allowable percent error in the measurement of the slant height $\ell$ that will ensure that the calculation of the lateral surface area $A = \pi \ell (a + b)$ is no more than 10% off.