Recall from Calc 1:

- If \( \lim_{x \to c^+} f(x) = L \) and \( \lim_{x \to c^-} f(x) = L \), then . . .

AND

- If \( \lim_{x \to c^+} f(x) \neq \lim_{x \to c^-} f(x) \), then . . .

Informal Def: We say \( \lim_{(x,y) \to (a,b)} f(x, y) = L \) if . . .

Note: If we want to show that \( \lim_{(x,y) \to (a,b)} f(x, y) \) Does Not Exist (DNE), then we must . . .

Ex 1: Show that \( \lim_{(x,y) \to (0,0)} \frac{7xy}{x^2 + y^2} \) does not exist.
Ex 2: Show that \( \lim_{(x,y) \to (0,0)} \frac{xy^2}{x^2 + y^4} \) does not exist.

Ex 3: Try to show that \( \lim_{(x,y) \to (0,0)} \frac{3x^2y}{x^2 + y^2} \) DOES exist.

Recall: To convert the point \((x, y)\) from rectangular coordinates to the polar coordinate point \((r, \theta)\), we use the formulas ...
Ex 4: Use polar coordinates to show that \( \lim_{{(x,y) \to (0,0)}} \frac{3x^2 y}{x^2 + y^2} \) DOES exist.

Ex 5: Use polar coordinates to show that \( \lim_{{(x,y) \to (0,0)}} \frac{xy}{\sqrt{x^2 + y^2}} \) exists.

WARNING!
Tip!!! A **shortcut** for computing limits is to use . . .

**Ex 6:** Use the shortcut to compute the following limits.

(a) \[ \lim_{{(x,y) \to (1,1)}} \frac{7xy^2}{{x^2 + y^2}} \]

(b) \[ \lim_{{(x,y) \to (1,1)}} \frac{7xy^2}{{x^2 - y^2}} \]

(c) \[ \lim_{{(x,y) \to (0,0)}} \frac{7xy^2}{{x^2 + y^2}} \]

**Summary:**

- To show a limit DNE . . .
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- To show a limit EXISTS . . .
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Recall from Calc 1: We say a function $f(x)$ is **continuous** at $x = c$ if

(a)

(b)

(c)

**Def:** We say a function $f(x, y)$ is **continuous** at $(x, y) = (a, b)$ if

(a)

(b)

(c)

**Ex 7:** Determine the set of points at which the function is continuous.

(a) $f(x, y) = \begin{cases} \frac{3x^2y}{x^2+y^2} & \text{if } (x, y) \neq (0, 0) \\ 1 & \text{if } (x, y) = (0, 0) \end{cases}$
(b) \( f(x, y) = \begin{cases} \frac{xy}{\sqrt{x^2 + y^2}} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases} \)