**Tangent Vectors and Normal Vectors**

**Ex 1:** Consider the curve $C$ given by $\mathbf{r}(t) = \langle t^2, t \rangle$.

(a) Find another vector-valued function $\mathbf{p}(s)$ that also traces out $C$ with the same orientation as is given by $\mathbf{r}$.

(b) Find the tangent vectors to $C$ at the point $(9, 3)$ using $\mathbf{r}$ and $\mathbf{p}$.

*Hint:* After computing $\mathbf{r}'$ and $\mathbf{p}'$, you will need to find $t_0$ and $s_0$ such that $\mathbf{r}(t_0) = \langle 9, 3 \rangle$ and $\mathbf{p}(s_0) = \langle 9, 3 \rangle$.

(c) Compare and contrast $\mathbf{r}'(t_0)$ and $\mathbf{p}'(s_0)$.

**Def:** Let $C$ be a smooth curve represented by $\mathbf{r}$ on an open interval $I$. The **unit tangent vector** $T(t)$ is given by ...
Ex 2: Find the unit tangent vector to the curve $C$ in Example 1 at the point $(9, 3)$.

Def: Let $C$ be a smooth curve represented by $\mathbf{r}$ on an open interval $I$. If the derivative of the unit tangent vector $T'(t) \neq 0$, then the principal unit normal vector $N(t)$ is given by . . .

Thm 1:

Proof:

Thm 2: $N(t)$ always . . .
Ex 3: Find the principal unit normal vector to the curve $C$ from Example 1 at the point $(9,3)$ on the curve.

**Shortcut for finding $N(t)$:**
If $C$ is a curve *in the plane*, and $\mathbf{T}(t) = \langle x(t), y(t) \rangle$, then . . .

**WARNING!!**

Ex 4: Use the shortcut to find the principal unit normal vector to the curve $C$ from Example 1 at the point $(9,3)$ on the curve.
Ex 5: Suppose an object has acceleration \( \mathbf{a} \).

- Using the same initial point, sketch \( \mathbf{a} \), the principal unit normal vector \( \mathbf{N} \), and the principal unit tangent vector \( \mathbf{T} \).
- Next sketch \( \text{proj}_\mathbf{N} \mathbf{a} \) and \( \text{proj}_\mathbf{T} \mathbf{a} \).
- Finally, write out the formulas for \( \text{proj}_\mathbf{N} \mathbf{a} \) and \( \text{proj}_\mathbf{T} \mathbf{a} \) and simplify them.

Def: The **tangential** \( a_T \) and **normal** \( a_N \) components of acceleration are ...
Ex 6: Given a position vector $\mathbf{r}(t) = (t^2, t, t)$, use both the definition of the principal unit normal vector and the above alternate method for finding the principal unit normal vector. Which is easier?