**FIRST ORDER LINEAR ODEs**

**Def:** A first order linear ODE is an equation that can be written as . . .

**Ex 1:** Are the following first order ODEs linear?

(a) \( \frac{dy}{dx} + y^3 = 0 \)

(b) \( \frac{dy}{dx} + y = x^3 \)

**Motivating Example:** Suppose \( x \frac{dy}{dx} + y = x^3 \).

**General Example:** Suppose \( \frac{dy}{dx} + P(x)y = Q(x) \).

Multiply BOTH sides by:

Integrate:
**Linear First Order:** The solution to

\[ \frac{dy}{dx} + P(x)y = Q(x) \]

is

**Ex 1:** Solve the ODE \( x \frac{dy}{dx} + 2y = x^{-3} \)

**Ex 2:** Solve the IVP \( \frac{dy}{dx} + \frac{3y}{x} + 2 = 3x, \quad y(1) = 1 \)

*Hint: Since we start at time \( x = 1 \), assume \( x > 0 \).*
Ex 3: Population of a certain Fish in LaVerne Lake reproduce with at a rate that is equal to the 0.8 times the current population of the fish, where \( t \) is measured in days. Additionally, these fish are harvested at a constant rate of 100 fish per day. Write and solve a DE which solves this situation.

(b) What will happen with an initial population of 110 fish? What about 1000?

Note: Sometimes \( P(x) \) or \( e^{\int P(u) \, du}Q(x) \) cannot be integrated: then the solution to \( \frac{dy}{dx} + P(x)y = Q(x) \)
(expressed as a definite integral) is

\[
y(x) = \frac{\int_0^x Q(u)e^{\int_0^w P(w) \, dw} \, du + C}{e^{\int_0^x P(u) \, du}}
\]