Vectors in the Plane and in Space

Def: A directed line segment $\overrightarrow{PQ}$ is ...

Def: A vector is a ...

Ex 1: Sketch the points $P = (0, 0)$, $Q = (1, 2)$, $R = (1, -1)$, and $S = (2, 1)$.

Which directed segments give equal vectors?

Def: A vector is in standard position (or standard form) if ...

Note:
Ex 2: Write the vector $\overrightarrow{RS}$ in standard form, where $R$ and $S$ are as in Example 1.

In general to express the directed line segment $\overrightarrow{AB}$ as vector in standard form, where $A = (a_1, a_2, a_3)$ and $B = (b_1, b_2, b_3)$ . . .

**Def:** The **length** (a.k.a. norm or magnitude) of a vector $\overrightarrow{PQ}$ in the plane, where $P = (x_1, y_1)$ and $Q = (x_2, y_2)$ is given by . . .

**Def:** The **length** (a.k.a. norm or magnitude) of a vector $\overrightarrow{PQ}$ in space, where $P = (x_1, y_1, z_1)$ and $Q = (x_2, y_2, z_2)$ is given by . . .

Ex 3: Compute the lengths of $w = \langle 2, -1 \rangle$ and $v = \langle 1, 3, -2 \rangle$.

Side Note 1: (Sphere of radius $r$ in 3 space centered at $(0, 0, 0)$

Side Note: (Sphere of radius $r$ in 3 space centered at $(x_0, y_0, z_0)$
Ex 4: Find the center and radius of the sphere given by $x^2 + 2x + y^2 - 6y + z^2 = 6$

Def: A three-dimensional coordinate system $x, y, z$ is right-handed if ... 

Ex 5: Plot the points $A = (1, 3, -2)$, $B = (-1, 2, 4)$, and $C = (1, 0, 2)$ in a right handed coordinate system.

Ex 6: Sketch the vectors $\mathbf{v} = \langle 1, 3, -2 \rangle$ and $\mathbf{w} = \langle 2, -1 \rangle$.

Note: The zero vector:

Def: A unit vector is ...
Def: **Scalar multiplication:** if \( \vec{v} = \langle v_1, v_2, v_3 \rangle \) and \( c \) is a real number (a scalar). Then

\[
c \vec{v} =
\]

**Def:** We say two vectors \( \vec{v} \) and \( \vec{w} \) are parallel if . . .

We say that two vectors \( \vec{v} \) and \( \vec{w} \) are in the same direction if . . .

**- To find a unit vector in the same direction as a non-zero vector \( \vec{v} \) . . .

**Ex 7:** Find unit vectors in the same direction as \( \vec{v} = \langle 1, 3, -2 \rangle \) and \( \vec{w} = \langle 2, -1 \rangle \).

**- The unit vector that makes an angle \( \theta \) with the positive \( x \)-axis is:

**Ex 8:** Find a vector of length 7 in the plane that makes an angle of 45° with the positive \( x \)-axis.
Ex 9: Find a vector of length 1/2 in the plane that makes an angle of 120° with the positive x-axis.

**Vector Operations**

Vector addition (seen algebraically and geometrically):
If \( \mathbf{u} \) and \( \mathbf{v} \) are vectors in standard position, then \( \mathbf{u} + \mathbf{v} = \)
Geometrically:

Note: \( \mathbf{u} - \mathbf{v} \) geometrically is

**Ex 10:** Suppose \( \mathbf{u} = \langle -1, 3 \rangle \) and \( \mathbf{v} = \langle 2, 4 \rangle \). Compute \( \mathbf{u} + \mathbf{v} \) algebraically and geometrically.

**Ex 11:** (ALGEBRA) Solve for \( x \)

(a) \( 5x + 3y = \langle -10, 4 \rangle \) where \( y = \langle 2, 3 \rangle \).
(b) \(6x - 2y = 0\) where \(y = (1, 2, 3)\).

PHYSICS: Forces are typically expressed as vectors. Two or more forces can be added and their net result computed as a vector sum. If an object is not moving, the sum of the force vectors is the zero vector. (Note: in physics, in the metric system, masses need to be converted to forces using \(F=ma\)).

**Ex 12:** Three forces with magnitudes 75 lbs, 100 lbs, and 125 lbs act on an object at angles 30°, 45°, and 120° with the positive \(x\)-axis, respectively. Find the direction and magnitude of the resultant of these forces.
Ex 13: Use the figure to determine the tension in each cable supporting the given load.

Ex 14: Use the figure to determine the tension in each cable, given that the tension on AB is 630lb.