Vectors in the Plane and in Space

Def: A vector is . . .

Def: A directed line segment $\overrightarrow{PQ}$ is . . .

Def: Two vectors are equal if . . .

Ex 1: Sketch the points $P = (0, 0)$, $Q = (1, 2)$, $R = (1, -1)$, and $S = (2, 1)$.

Observe:

WARNING!

Def: A vector is in standard position (or standard form) if . . .
Def: If a vector \( \mathbf{v} \) in the plane is in standard position with its terminal point at \((v_1, v_2)\), then the component form of \( \mathbf{v} \) is . . .

Likewise, if a vector \( \mathbf{w} \) in space is in standard position with its terminal point at \((w_1, w_2, w_3)\), then the component form of \( \mathbf{w} \) is . . .

Ex 2: Write the vector \( \overrightarrow{RS} \) in component form, where \( R \) and \( S \) are as in Example 1.

Def: A three-dimensional coordinate system is right-handed if ...

Ex 3a: Plot the points \( A = (1, 3, -2) \), \( B = (-1, 2, 4) \), and \( B = (1, 0, 2) \) on a right handed coordinate system.

Ex 3b: Sketch the vectors \( \mathbf{v} = \langle 1, 3, -2 \rangle \) and \( \mathbf{w} = \langle 2, -1 \rangle \).
Note:

Ex 4: Sketch the vectors $(0, 0)$ and $(0, 0, 0)$. 
Def: The **length** of a vector $\overrightarrow{PQ}$ in the plane, where $P = (x_1, y_1)$ and $Q = (x_2, y_2)$ is given by . . .

Def: The **length** of a vector $\overrightarrow{PQ}$ in space, where $P = (x_1, y_1, z_1)$ and $Q = (x_2, y_2, z_2)$ is given by . . .

**Ex 5:** Compute the lengths of $w = \langle 2, -1 \rangle$ and $v = \langle 1, 3, -2 \rangle$.

Def: A **unit vector** is . . .

Def: The **Standard Unit Vectors in the Plane** are . . .

Def: The **Standard Unit Vectors in Space** are . . .

**Ex 6:** Express the vectors $v = \langle 1, 3, -2 \rangle$ and $w = \langle 2, -1 \rangle$ using unit vector notation.
Def: We say two vectors \( \mathbf{v} \) and \( \mathbf{w} \) are parallel if . . .

Ex 7: Find a unit vector \( \mathbf{u} \) in the plane that is parallel to \( \mathbf{w} = \langle 2, -1 \rangle \) and a unit vector \( \mathbf{U} \) in space that is parallel to \( \mathbf{v} = \langle 1, 3, -2 \rangle \).

Important Note: The unit vector in the plane that makes an angle of \( \theta \) with the positive \( x \)-axis is given by . . .

Ex 8: Find a vector of length 7 in the plane that makes an angle of 45° with the positive \( x \)-axis.

Ex 9: Find a vector of length 10 in the plane that makes an angle of 120° with the positive \( x \)-axis.
VECTOR OPERATIONS

Basic Operations (seen algebraically and geometrically):
If \( \mathbf{u} = \langle u_1, u_2 \rangle \) and \( \mathbf{v} = \langle v_1, v_2 \rangle \), then

1. \( \mathbf{u} + \mathbf{v} = \)

2. For any scalar \( k \), \( k\mathbf{u} = \)

3. \( -\mathbf{v} = \)

Ex 10: Suppose \( \mathbf{u} = \langle -10, 4 \rangle \) and \( \mathbf{v} = \langle 2, 3 \rangle \). Compute \( 2\mathbf{u} - \mathbf{v} \), then draw a sketch that shows this operation geometrically.
**Ex 11:** Three forces with magnitudes 75 lbs, 100 lbs, and 125 lbs act on an object at angles $30^\circ$, $45^\circ$, and $120^\circ$ with the positive $x$-axis, respectively. Find the direction and magnitude of the resultant of these forces.

**Ex 13:** Three forces with magnitudes 300 Newtons, 180 Newtons, and 250 Newtons act on an object at angles $-30^\circ$, $45^\circ$, and $135^\circ$ with the positive $x$-axis, respectively. Find the direction and magnitude of the resultant of these forces.
Ex 14: Ropes 3 meters and 5 meters in length are fastened to a holiday decoration that is suspended over a town square. The decoration has a mass of 5 kg. The ropes, fastened at different heights, make angles of $52^\circ$ and $40^\circ$ with the horizontal. Find the tension in each wire.

Ex 15: Use the figure to determine the tension in each cable supporting the given load.
Ex 16: Use the figure to determine the tension in each cable, given that the tension on AB is 630lb.