HW 13.4: 1, 3, 11, 15, 17, 25, 33, 35, 45
HW 13.5: 1, 5, 7, 19, 23, 53, 55, 56

**Differentials**

Definition of $\Delta$:
$\Delta x$ represents:

$\Delta y$ represents:

If $z = f(x, y)$, then $\Delta z$ represents:

**Def:** Let $z = f(x, y)$. The **total differential** of $z$ is given by . . .

**Def:** Let $z = f(x, y)$ The tangent plane approximation of $(x, y)$ is given by

$T(x, y) =$

**Informal Def:** We say the function $z = f(x, y)$ is **differentiable** at $(a, b)$ if . . .

**Ex 3a:** Use differentials to approximate: $\sqrt{16.04} \sqrt{7.99}$
Application: Error Analysis
Definition of relative change of quantity $Q$:

Use the following fact that:

**Ex 3:** A manufacturer makes cylindrical cans. Estimate the error in volume if an error of 5% is made in the radius and an error of 1% is made in the height.

(b) If the errors made in the height and radius are approximately the same, what error could we have in these to produce an error in volume less than 1%?

**Ex 4:** Electrical power $P$ is given by $P = E^2/R$, where $E$ is the voltage and $R$ is the resistance. The possible percentage errors in measuring $E$ and $R$ are 2% and 3%, respectively. Approximate the maximum percentage error in calculating the power. Then approximate the maximum propagated error in calculating power if 200 volts is applied to a 4000-ohm resistor.
**Section 13.5**

**Chain Rules for Functions of Several Variables**

Recall from Calc 1: The chain rule says that

\[
\frac{d}{dx} [f(g(x))] = \ldots
\]

**Chain Rule for** \( f(x(t), y(t)) \)

If \( z = f(x(t), y(t)) \), then \( \frac{dz}{dt} = \)

Notation: We also write this as \( \frac{d}{dt} [f(x(t), y(t))] \)

Notation: if \( r(t) = x(t)i + y(t)j \) and \( f(x, y) \) is a function, then \( f(r(t)) \) makes sense, and \( \frac{d}{dt} f(r(t)) = \)

(This form looks nicer)

**Ex 1:** Let \( z = f(x, y) = x^2 e^y \) and \( r(t) = (\sin t, t^3) \). Find \( \frac{dz}{dt} \).

Observe:
Application of Chain Rule: Related Rates

Ex 2: A tree trunk may be considered a circular cylinder. Suppose the diameter of the trunk increases 1 inch per year and the height of the trunk increases 6 inches per year. How fast is the volume of wood in the trunk increasing when it is 100 inches high and 5 inches in diameter?
Chain Rule for $z = f(x(s, t), y(s, t))$:
If $z = f(x(s, t), y(s, t))$ and (here $x$ and $y$ each depend on $s$ and $t$) then

$\frac{\partial z}{\partial s} =$

$\frac{\partial z}{\partial t} =$

Ex 3: Let $f(x, y) = x \ln y$, $x = s^2 + t^2$, and $y = s^2 - t^2$.

(a) Compute $\frac{\partial f}{\partial s}$.

(b) Compute $f_t$.

Chain rule for $w = f(x(t), y(t), z(t))$:

$\frac{d}{dt}[f(x(t), y(t), z(t))] =$

Again, if $w = f(x, y, z)$ and $r(t) = \langle x(t), y(t), z(t) \rangle$, then

$\frac{d}{dt}[f(r(t))] =$
Lastly, \( w = f(x(s, t), y(s, t), z(s, t)) \):
\[
\frac{\partial}{\partial s} [f(x(s, t), y(s, t), z(s, t))] =
\]
\[
\frac{\partial}{\partial t} [f(x(s, t), y(s, t), z(s, t))] =
\]

Other chain rules exist for functions of more than 3 variables.

This is the last MATH 231 topic.