
**Tangent Planes and Normal Lines to Surfaces**

There are 3 Ways to Describe a Surface:

1. 

2. 

3. 

It is method 3 that we need in this section.

**Ex 0:** (a) Express $x^2 + y^2 = 1 + z^2$ as a level surface for a function of 3 variables

(b) Express the graph of $f(x, y) = x^2 + y^2 - 1$ as a level surface for a function of 3 variables

**Def:** Let $S$ be a smooth level surface given by $f(x, y, z) = K$. A **normal vector** to $S$ at $(x_0, y_0, z_0)$ (which must be on $S$) is given by . . .

**Def:** Let $S$ be a smooth level surface given by $f(x, y, z) = K$. The **normal line** to $S$ thru $(x_0, y_0, z_0)$ is given by . . .
**Def:** Let $S$ be a smooth level surface given by $f(x, y, z) = K$. The **tangent plane** to $S$ at $(x_0, y_0, z_0)$ is . . .

***To find the normal vector to a surface at $(x_0, y_0, z_0)$, first describe the surface in question as a level surface of a function of 3 variables $F(x, y, z)$. Then $\nabla f(x_0, y_0, z_0)$ (if it is nonzero) is a normal vector to the surface.

**Ex 0:** Let $z = x^2 + y^2$.

(a) Find a normal vector to the surface at $(1, -1, 2)$.

(b) Find a set of parametric equations for the normal line at $(1, -1, 2)$.

(c) Find an equation for the tangent plane at $(1, -1, 2)$.
Ex 1: Let $f(x, y) = x^3 - 3x^2y + y^2$.

(a) Find the tangent plane to the graph of $f$ at the point $(1, -1, 5)$.

(b) Find a set of parametric equations for the normal line to the graph of $f$ at $(1, -1, 5)$.

Ex 2: Find the equation of the tangent plane to the surface given by $z^2 = x^2 + y^2$ at the point $P = (1, 2, \sqrt{5})$. 
Ex 3: Consider the surfaces given by $z = x^2 + y^2$ and $x + y + 6z = 33$. Observe that the point $P = (1, 2, 5)$ is on the curve of intersection of these surfaces.

Find the angle of intersection between the surfaces at the point $P$. 