**Vector-Valued Functions**

**HW 12.1:** 9, 13, 17, 21-24, 27, 31, 35, 39, 41, 49, 53, 55, 57, 59, 63, 67

**Def:** A space curve or parameterized curve is ...

A special type of space curve that we have already seen is ...

**Def:** A vector-valued function is a function of the form ...

**To plot vector valued functions, we treat the vectors as points (the terminal points of the vectors in standard position)!!

**Goals:** 1.

2.

**HINT to Plot:** find relationships between $x, y, z$!!

**Ex 1:** Sketch the curve traced out by the vector-valued function $\mathbf{r}(t) = \langle 1, t, t^2 \rangle$. Include the orientation of the curve.

**NOTE:** As before, parameterizations of a curve are
Ex 2a: Sketch the curve traced out by \( \mathbf{r}(t) = \langle 2 \cos t, 0, 2 \sin t \rangle \). Include the orientation of the curve.

Ex 2b: Sketch the curve traced out by \( \mathbf{r}(t) = \langle 2 \cos t, t, 2 \sin t \rangle \). Include the orientation of the curve. This curve is called a ________________________________

Ex 3a: Sketch the curve traced out by \( \mathbf{r}(t) = \langle t, \sin t, t^2 \rangle \). Include the orientation of the curve.
**Ex 3b:** Sketch the curve traced out by \( r(t) = (t^2, t^4, 0) \).

*** In the plane: *** to parameterize the graph of \( y = f(x) \) (from left to right) use ....

NOTES: (1) If \( r(t) \) traces out a curve, then \( r(-t) \) [replace \( t \) by \(-t\)] will trace the same curve ________________.

(2) If \( r(t) \) traces out a curve, then \( r(2t) \) [replace \( t \) by \(2t\)] will trace the same curve ________________.

(3) If \( r(t) \) traces out a curve, then \( r(t - c) \) [replace \( t \) by \( t - c\)] will trace the same curve ________________.

**Ex 4a:** Find two parameterizations for \( y = x^3 \), for \( 0 \leq x \leq 2 \) one from left to right, and one from right to left.
*** In general: to parameterize a curve, try setting $t$ equal to one of: $x$, $y$, or $z$. If circles (ellipses) are present (e.g. $\frac{x^2}{a^2} + \frac{y^2}{b^2}$) then try $x = a \cos t$ and $y = b \sin t$.

**Ex 4b:** Find two parameterizations for the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$. (NOTE: DO not use $\theta$ as a parameter –BAD BOOK!)

**Ex 5:** Find a vector-valued function to represent the curve of intersection between surfaces given by $x^2 + y^2 + z^2 = 16$ and $x + y = 4$. (Assume $x$, $y$, and $z$ are all positive.)

**Ex 6:** Use a vector-valued function to represent the curve of intersection between the surfaces given by $z = x^2 - y^2$ and $x^2 + y^2 = 1$. 

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